1. $\left(\underset{a_{n-1}}{x-1},\ \underset{a_n}{2x},\ \underset{a_{n+1}}{4x+3}\right)$ – ciąg geom.

$a_n^2 = a_{n-1} \cdot a_{n+1}$

$(2x)^2 = (x-1)(4x+3)$ 1 pkt

$\cancel{4x^2} = \cancel{4x^2} + 3x - 4x - 3$

$\boxed{x = -3}$ 1 pkt

Punktacja

0 - 6 ndst

6,5 - 7,5 dop

8 - 10,5 dst

11 - 14 db

14,5 - 16 bdb

2.

$$a_n = 3^{n+1} + 3^n + (-3)^{n-1}$$

$$a_1 = 3^2 + 3^1 + (-3)^0 = 9 + 3 + 1 = 13$$

1 pkt 1 pkt

3.

$$-11, -5, 1, \ldots$$

$a_1 = -11$ 0,5 pkt $\quad r = a_2 - a_1 = -5 - (-11) = 6$ 0,5 pkt

$$S_n = \frac{a_1 + a_n}{2} \cdot n$$

$$a_n = a_1 + (n-1) \cdot r$$

$$S_n = \frac{2a_1 + (n-1)r}{2} \cdot n$$

$$S_8 = \frac{2 \cdot (-11) + 7 \cdot 6}{2} \cdot 8$$ 0,5 pkt

$$S_8 = \frac{-22 + 42}{2} \cdot 8$$

$$S_8 = 80$$ 0,5 pkt

4. $a_n = n^2 - n - 12 \quad n \in \mathbb{N} \quad a_n < 0$

$n \geq 1$

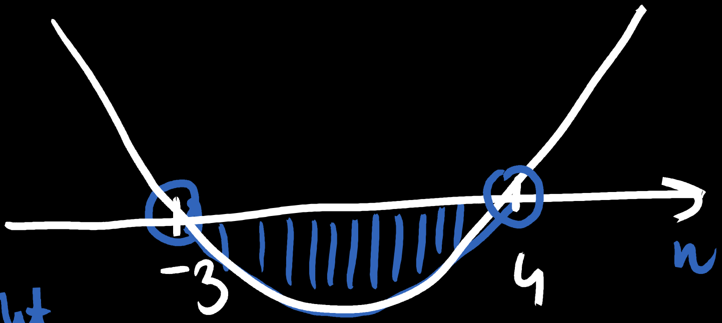
$n^2 - n - 12 < 0$ 0,5 pkt

$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-12) = 1 + 48 = 49$

$\sqrt{\Delta} = 7$

$n_1 = \frac{1-7}{2} = -3$

$n_2 = \frac{1+7}{2} = 4$ 0,5 pkt



0,5 pkt

$n \in (-3, 4)$

$n \in \{1, 2, 3\}$ 0,5 pkt

$a_1, a_2, a_3 < 0$

5.

$a_n = 4n - 5 \qquad a_{n+1} = 4(n+1) - 5 = 4n + 4 - 5 = 4n - 1$

0,5 pkt

$a_{n+1} - a_n = 4n - 1 - (4n - 5) = \cancel{4n} - 1 - \cancel{4n} + 5 = 4 > 0$

0,5 pkt

0,5 pkt

Ciąg arytmetyczny 0,25 pkt

Ciąg rosnący 0,25 pkt

6.

$K_0 = 40000$ zł

$n = 3 \cdot 2 = 6$ 0,5pkt

$p = 6\% : 2 = 3\%$

0,5pkt

$K_n = K_0 \cdot \left(1 + \frac{p}{100}\right)^n$

$K_n = 40\,000 \left(1 + \frac{3}{100}\right)^6$ 0,5pkt

$K_n = 40\,000 \; (1{,}03)^6$

$K_n = 47\,762{,}09$ zł 0,5pkt

7. (a, b, c) – c. geom. mal $(a, b, c-4)$ – c. arytm.

$$\begin{cases} 1)\ a+b+c = 13 \\ 2)\ b^2 = ac \\ 3)\ b = \frac{a+c-4}{2} \end{cases}$$

0,5 pkt

$\Rightarrow 2b = a+c-4 \Rightarrow a+c = \boxed{2b+4}$

$a+c = 10$

do 1) $2b+4+b = 13$

$3b = 9$

$\boxed{b=3}$ 0,5 pkt

do 2) $9 = a\cdot c$

$$\begin{cases} 9 = a\cdot c \\ \boxed{a = 10-c} \end{cases}$$

$9 = (10-c)\cdot c$

$9 = 10c - c^2$

$c^2 - 10c + 9 = 0$

$\Delta = 100 - 36 = 64$

$\sqrt{\Delta} = 8$

$c_1 = \frac{10-8}{2} = 1$

$c_2 = \frac{10+8}{2} = 9$

0,5 pkt

$a_1 = 9$

$a_2 = 1$

0,5 pkt

$$\boxed{\begin{cases} a = 9 \\ b = 3 \\ c = 1 \end{cases}} \qquad \begin{cases} a = 1 \\ b = 3 \\ c = 9 \end{cases} \text{ sprz.}$$

8. $$\lim_{n \to \infty} \frac{12n^2 - 5n + 3}{4n^2 + 2n - 13} =$$

$$= \lim_{n \to \infty} \frac{\frac{12n^2}{n^2} - \frac{5n}{n^2} + \frac{3}{n^2}}{\frac{4n^2}{n^2} + \frac{2n}{n^2} - \frac{13}{n^2}}$$ 0,5 pkt

$$= \lim_{n \to \infty} \frac{\overset{\nearrow 12}{12} - \overset{\nearrow 0}{\frac{5}{n}} + \overset{\nearrow 0}{\frac{3}{n^2}}}{\underset{\swarrow 4}{4} + \underset{\swarrow 0}{\frac{2}{n}} - \underset{\searrow 0}{\frac{13}{n^2}}} =$$ 0,5 pkt

$$= \frac{12 - 0 + 0}{4 + 0 - 0} = \frac{12}{4} = 3$$ 1 pkt