$$1B \quad (\underset{a_{n-1}}{7+x}, \underset{a_n}{9+2x}, \underset{a_{n+1}}{11+3x}) - \text{ciąg arytmetyczny}$$

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

$$a_n^2 = a_{n-1} \cdot a_{n+1}$$

$$9+2x = \frac{7+x+11+3x}{2} \quad / \cdot 2$$

$$2(9+2x) = 18+4x$$

$$\cancel{18}+\cancel{4x} = \cancel{18}+\cancel{4x}$$

$$0 = 0$$

$$\boxed{x \in \mathbb{R}}$$

2B $\begin{cases} a_2 = 7 \\ a_4 = 28 \end{cases}$

$\boxed{a_n = ?}$ c. geom.

$$a_n = a_1 \cdot q^{n-1}$$

$a_1 = ?$

$q = ?$

$$a_2 \cdot q^2 = a_4$$

$$7 \cdot q^2 = 28 \quad / :7$$

$$q^2 = 4 \quad / \sqrt{}$$

1° $\boxed{q = 2}$ 2° $\boxed{q = -2}$

$$a_2 = a_1 \cdot q$$

1° $7 = a_1 \cdot 2 \quad / :2$

$\begin{cases} \frac{7}{2} = a_1 \\ 2 = q \end{cases}$

$\boxed{a_n = \frac{7}{2} \cdot 2^{n-1}}$

2° $7 = a_1 \cdot (-2)$

$\begin{cases} -\frac{7}{2} = a_1 \\ -2 = q \end{cases}$

$\boxed{a_n = -\frac{7}{2} \cdot (-2)^{n-1}}$

3B $\quad -7, -4, -1, \ldots$ – c. arytm.

$-7 \xrightarrow{+r} -4$

$$a_n = a_1 + (n-1)\cdot r$$

$a_{50} = ?$

$a_1 = -7$

$n = 50$

$r = 3$

$a_1 + r = a_2$

$-7 + r = -4$

$r = 3$

$a_{50} = -7 + (50-1)\cdot 3$

$a_{50} = -7 + 147$

$\underline{a_{50} = 140}$

4B $a_n = \frac{n+1}{n+2}$ $\quad n \in \mathbb{N}$ $\quad$ arytm? $\quad$ monot?

$$a_{n+1} = \frac{n+1+1}{n+1+2} = \frac{n+2}{n+3}$$

$$a_{n+1} - a_n > 0 \quad \nearrow$$
$$< 0 \quad \searrow$$
$$= 0 \quad \rightarrow$$

$$(n+2)^2$$

$$a_{n+1} - a_n = \frac{n+2}{n+3} - \frac{n+1}{n+2} = \frac{(n+2)\cdot(n+2)}{(n+3)(n+2)} - \frac{(n+1)\cdot(n+3)}{(n+3)(n+2)} =$$

$$= \frac{n^2+2n+2n+4-(n^2+3n+n+3)}{(n+3)(n+2)} = \frac{\cancel{n^2}+\cancel{4n}+4-\cancel{n^2}-\cancel{4n}-3}{(n+3)(n+2)} =$$

$$= \frac{1 \quad {>0}}{(n+3)(n+2) \quad {>0}} > 0$$

Ciąg nie jest arytmetyczny

Ciąg rosnący

5β $a_n = n^2 - 6n - 12$ $a_n = -21$

$-21 = n^2 - 6n - 12$

$0 = n^2 - 6n + 9$

$\Delta = (-6)^2 - 4 \cdot 1 \cdot 9$

$\Delta = 36 - 36 = 0$

$x_0 = \frac{-b}{2a} = \frac{6}{2} = 3$

$a_3 = -21$

6B $K_n = K_0 \cdot \left(1 + \frac{p}{100}\right)^n$

$K_0 = 2000$

$p = 4\% : 4 = 1\%$

$n = 2 \cdot 4 = 8$

kap. co kwartał

$K_n = 2000 \cdot \left(1 + \frac{1}{100}\right)^8$

$K_n = 2000\,(1{,}01)^8$

$K_n = 2164$ zł

7B $\underbrace{5,\ \overbrace{a,\ b}}_{}$, 30 — c. arytm.

c. geom.

geom.

$$a_n^2 = a_{n-1} \cdot a_{n+1}$$

$$a^2 = 5b$$

$$(2b-30)^2 = 5b$$

$$(x-y)^2 = x^2-2xy+y^2$$

$$4b^2 - 120b + 900 = 5b$$

$$4b^2 - 125b + 900 = 0$$

arytm.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

$$b = \frac{a+30}{2} \quad / \cdot 2$$

$$2b = a + 30$$

$$2b - 30 = a$$

7Bc.d $4b^2 - 125b + 900 = 0$

$$\boxed{a = 2b - 30}$$

$\Delta = b^2 - 4ac$

$\Delta = (-125)^2 - 4 \cdot 4 \cdot 900$

$\Delta = 15625 - 14400$

$\Delta = 1225$

$\sqrt{\Delta} = 35$

$$\begin{cases} a = -7\frac{1}{2} \\ b = 11\frac{1}{4} \end{cases} \qquad \begin{cases} a = 10 \\ b = 20 \end{cases}$$

$b_1 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{125-35}{8} = \dfrac{90}{8} = \dfrac{45}{4} = 11\frac{1}{4}$

$b_2 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{125+35}{8} = \dfrac{160}{8} = 20$

$a_1 = 2 \cdot \dfrac{45}{4} - 30 = 22\frac{1}{2} - 30$

$= -7\frac{1}{2}$

$a_2 = 40 - 30 = 10$

8. $$a_n = \frac{3n^2 + 2n + 6}{6n^2 + 4n + 3}$$

$$\lim_{n \to \infty} \frac{2 - 7n^6}{3n^6 + 2n^2} = \frac{-7}{3}$$

$$\lim_{n \to \infty} \frac{3n^2 + 2n + 6}{6n^2 + 4n + 3} = \lim_{n \to \infty} \frac{\frac{3n^2}{n^2} + \frac{2n}{n^2} + \frac{6}{n^2}}{\frac{6n^2}{n^2} + \frac{4n}{n^2} + \frac{3}{n^2}} =$$

$$= \lim_{n \to \infty} \frac{\underbrace{3}_{\to 3} + \underbrace{\frac{2}{n}}_{\to 0} + \underbrace{\frac{6}{n^2}}_{\to 0}}{\underbrace{6}_{\to 6} + \underbrace{\frac{4}{n}}_{\to 0} + \underbrace{\frac{3}{n^2}}_{\to 0}} = \frac{3 + 0 + 0}{6 + 0 + 0} = \frac{3}{6} = \frac{1}{2}$$

1A $(\underset{a}{x-1},\ \underset{b}{2x+1},\ \underset{c}{4x})$ - ciąg geom.

$b^2 = ac$ $\qquad$ $a_n^2 = a_{n-1} \cdot a_{n+1}$

$(2x+1)^2 = (x-1) \cdot 4x$

$(a+b)^2 = a^2 + 2ab + b^2$

$4x^2 + 4x + 1 = 4x^2 - 4x$

$\cancel{4x^2} + 4x + 1 - \cancel{4x^2} + 4x = 0$

$8x + 1 = 0$

$8x = -1 \quad /:8$

$\boxed{x = -\frac{1}{8}}$

2A. $$a_n = 3^{n+1} + 3^n + (-3)^{n-1}$$

$$a \neq 0$$
$$a^0 = 1$$

$$a_1 = 3^{1+1} + 3^1 + (-3)^{1-1} = 3^2 + 3 + (-3)^0 = 9 + 3 + 1 =$$
$$= 13$$

$$a_3 = 3^{3+1} + 3^3 + (-3)^{3-1} = 3^4 + 27 + (-3)^2 =$$
$$= 81 + 27 + 9 = 117$$

4. $a_n = n^2 - 4n - 12$, $n \geq 1$ $\qquad a_n < 0$

$n \in \mathbb{N}$

$$n^2 - 4n - 12 < 0$$

$\Delta = b^2 - 4ac$

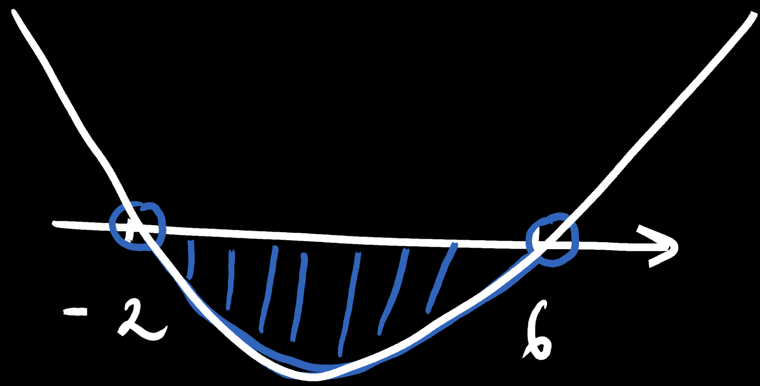
$\Delta = (-4)^2 - 4 \cdot 1 \cdot (-12)$

$\Delta = 16 + 48 = 64$

$\sqrt{\Delta} = 8$

$n_1 = \dfrac{-b - \sqrt{\Delta}}{2a} = \dfrac{4 - 8}{2} = -2$

$n_2 = \dfrac{-b + \sqrt{\Delta}}{2a} = \dfrac{4 + 8}{2} = 6$



$n \in (-2, 6) \cap \mathbb{N}$

$n \in \{1, 2, 3, 4, 5\}$

7A

$$\begin{cases} 1)\ a+b+c=13 \\ 2)\ b^2 = ac \\ 3)\ b = \dfrac{(a+c)-4}{2} \end{cases} \Rightarrow 2b = (a+c)-4 \Rightarrow 2b+4 = (a+c)$$

a, b, c – c. geom. (malejący)

$a, b, c-4 \leftarrow$ c. arytm.

do 1) $(a+c)+b = 13$

$2b+4+b = 13$

$3b+4 = 13$

$3b = 9$

$\boxed{b=3}$

do 2) $3^2 = a\cdot c$

$9 = a\cdot c$

$9 = a\cdot(10-a)$

$9 = 10a - a^2$

$a^2 - 10a + 9 = 0$

$a+c = 10$

$c = \boxed{10-a}$

7A c. d

$$a^2 - 10a + 9 = 0$$

$$\Delta = 100 - 36 = 64$$

$$\sqrt{\Delta} = 8$$

$$a_1 = \frac{10-8}{2} = 1$$

$$a_2 = \frac{10+8}{2} = 9$$

$$\begin{cases} a = 1 \\ b = 3 \\ c = 9 \end{cases}$$

$$c_1 = 10 - 1 = 9$$

$$c_2 = 10 - 9 = 1$$

$$\begin{cases} a = 9 \\ b = 3 \\ c = 1 \end{cases}$$