8. 1) $\underbrace{a} + b + \underbrace{c} = 21$

$a, b, c$ – c. arytm.

2) $b = \frac{a+c}{2} \,/\cdot 2 \Rightarrow \underbrace{a+c} = 2b$

$a, b-1, c+6$ – c. geom.

3) $(b-1)^2 = a\cdot(c+6)$

$x, y, z$

do 1) $\underbrace{a+c} + b = 21$

$2b + b = 21$

$3b = 21$

$\underbrace{b = 7}$

do 2) $a + c = 14$

do 3) $(7-1)^2 = a(c+6)$

$36 = a(c+6)$

$y = \sqrt{x \cdot z}$

$y^2 = xz$

Trzy liczby, których suma jest równa 21, tworzą ciąg arytmetyczny. Jeśli od drugiej z nich odejmiemy 1, a do trzeciej dodamy 6, to otrzymamy ciąg geometryczny. Wyznacz te liczby.

8.c.d

$$* \begin{cases} a + c = 14 \\ 36 = a(c+6) \end{cases} \Rightarrow \underline{c = 14 - a}$$

$$\begin{cases} a = 2 \\ b = 7 \\ c = 12 \end{cases} \qquad \begin{cases} a = 18 \\ b = 7 \\ c = -4 \end{cases}$$

$$36 = a(14 - a + 6)$$

$$36 = a(20 - a)$$

$$36 = 20a - a^2$$

$$a^2 - 20a + 36 = 0$$

$$\Delta = b^2 - 4ac \qquad a_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{20 - 16}{2} = 2 \qquad c_1 = 14 - 2 = 12$$

$$\Delta = (-20)^2 - 4 \cdot 1 \cdot 36$$

$$\Delta = 400 - 144 \qquad a_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{20 + 16}{2} = 18 \qquad c_2 = 14 - 18 = -4$$

$$\Delta = 256$$

$$\sqrt{\Delta} = 16$$

3. $\begin{cases} a_1 + a_5 = 68 \\ a_2 + a_6 = 136 \end{cases}$

$S_n = 2044 \qquad n = ?$

$$a_n = a_1 \cdot q^{n-1}$$

$$a_5 = a_1 \cdot q^4$$

$$a_2 = a_1 \cdot q$$

$$a_6 = a_1 \cdot q^5$$

$$\begin{cases} a_1 + a_1 \cdot q^4 = 68 \\ a_1 \cdot q + a_1 \cdot q^5 = 136 \end{cases}$$

$$* \begin{cases} a_1 \cdot (1+q^4) = 68 \,/ :(1+q^4) \Rightarrow a_1 = \underbrace{\dfrac{68}{1+q^4}}_{} = \dfrac{68}{1+2^4} = \dfrac{68}{1+16} = \\ \underset{\uparrow}{a_1} q\,(1+q^4) = 136 \end{cases}$$

$$= \frac{68}{17} = 4$$

$$\frac{68}{\cancel{(1+q^4)}} \cdot q \cdot \cancel{(1+q^4)} = 136$$

$$68q = 136$$

$$\underbrace{q = 2}_{}$$

$$\begin{cases} a_1 = 4 \\ q = 2 \end{cases}$$

3cd $a_1 = 4$ $n = ?$

$q = 2$

$S_n = 2044$

$$S_n = a_1 \frac{1 - q^n}{1 - q}$$

$$2044 = 4 \cdot \frac{1 - 2^n}{1 - 2} \quad / : 4$$

$$511 = \frac{1 - 2^n}{-1} \quad / \cdot (-1)$$

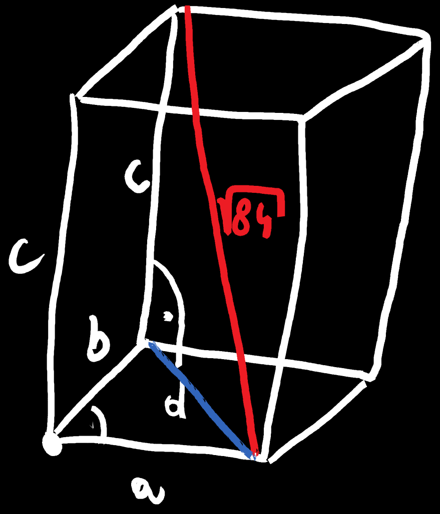
$$-511 = 1 - 2^n \quad / - 1$$

$$-512 = -2^n \quad / \cdot (-1)$$

$$512 = 2^n$$

$$9 = n$$

10. Długości trzech krawędzi prostopadłościanu wychodzących z tego samego wierzchołka tworzą ciąg geometryczny. Przekątna prostopadłościanu ma długość √84 cm, a jego objętość jest równa 64 cm3. Oblicz pole powierzchni całkowitej tego prostopadłościanu.



$$\boxed{a \xrightarrow{\cdot q} aq \xrightarrow{\cdot q} aq^2}$$

a, b, c – ciąg geom.

1) $b^2 = \underline{a \cdot c}$

$4^2 = a \cdot c$

$\underline{a \cdot c = 16}$

$a^2 + b^2 = \underbrace{d}^{}{}^2$

$c^2 + d^2 = \sqrt{84}^2$

2) $\underline{c^2 + a^2 + b^2 = 84} \Rightarrow c^2 + a^2 + 16 = 84$

$\underline{c^2 + a^2 = 68}$

$V = a \cdot b \cdot c$

3) $64 = abc \Rightarrow 64 = \underline{a \cdot c} \cdot b$

$64 = b^2 \cdot b$

$64 = b^3$

$\underline{b = 4}$

10 c.d.

$$\begin{cases} c^2+a^2=68 \\ a\cdot c=16 \quad /:a \Rightarrow \quad c=\frac{16}{a} \end{cases}$$

$$\left(\frac{16}{a}\right)^2+a^2=68$$

$$\frac{256}{a^2}+a^2=68 \quad /\cdot a^2$$

$$256+a^4=68a^2$$

$$a^4-68a^2+256=0$$

$$\boxed{a^2=t} \qquad t\geqslant 0$$

$$t^2-68t+256=0$$

$$\Delta=(-68)^2-4\cdot 256$$

$$\Delta=4624-1024$$

$$\Delta=3600$$

$$\sqrt{\Delta}=60$$

$$t_1=\frac{-b-\sqrt{\Delta}}{2a}=\frac{68-60}{2}=4$$

$$t_2=\frac{-b+\sqrt{\Delta}}{2a}=\frac{68+60}{2}=64$$

$$\begin{cases} a=2 \\ b=4 \\ c=8 \end{cases}$$

$a^2=4$

$a_1=2 \quad c_1=8$

~~$a_2=-2 \quad c_2=-8$~~

$a^2=64$

$a_3=8 \quad c_3=2$

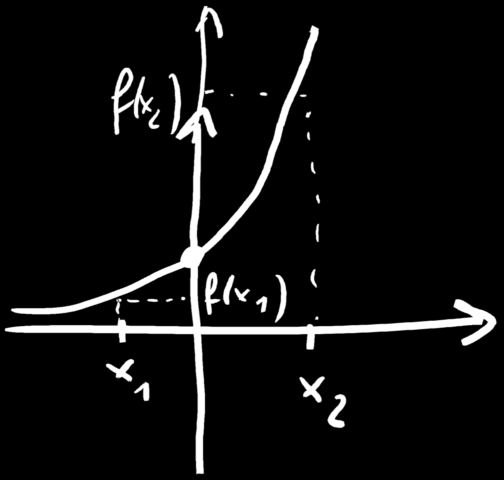
~~$a_4=-8 \quad c_4=-2$~~

11. Z: $a_n$ - c. arytm. $a_n \in \mathbb{C}$

$$\boxed{a_n = \frac{a_{n-1}+a_{n+1}}{2}}$$

T: $b_n = 2^{a_n}$ - c. geom. (?) $b_n^2 = b_{n-1}\cdot b_{n+1}$

$$a_n = \frac{a_{n-1}+a_{n+1}}{2} \quad /2^{?}$$



$$x = y$$

$$2^x = 2^y$$

$$2^{a_n} = 2^{\frac{a_{n-1}+a_{n+1}}{2}}$$

$$2^{a_n} = 2^{\frac{1}{2}(a_{n-1}+a_{n+1})} \quad /(\quad)^2$$

$$\left(a^{\frac{1}{2}}\right)^2 = a$$

$$\left(2^{a_n}\right)^2 = \left[2^{\frac{1}{2}(a_{n-1}+a_{n+1})}\right]^2$$

11. c. d

$$\left(2^{a_n}\right)^2 = 2^{a_{n-1}+a_{n+1}}$$

$$a^{n+m} = a^n \cdot a^m$$

$$\left(2^{a_n}\right)^2 = 2^{a_{n-1}} \cdot 2^{a_{n+1}}$$

$$b_n = 2^{a_n}$$

$$b_n^2 = b_{n-1} \cdot b_{n+1}$$

$$b_{n-1} = 2^{a_{n-1}}$$

$$b_{n+1} = 2^{a_{n+1}}$$

c. n. d