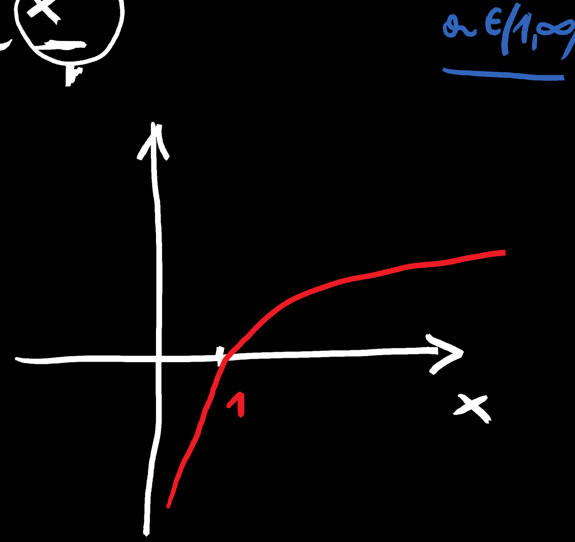
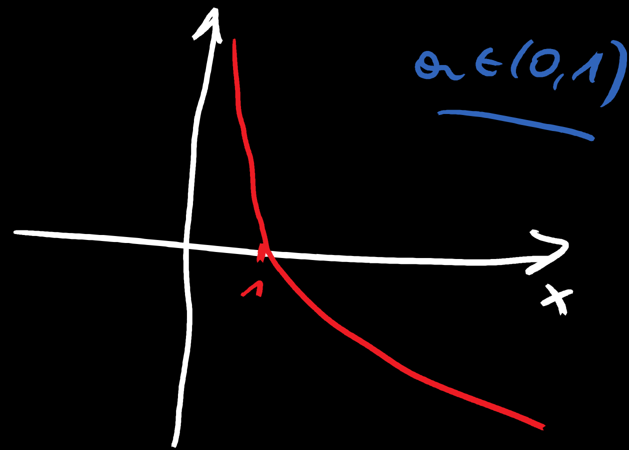
$$f(x) = \log_{\underbrace{x+1}_{a}}\underbrace{(4-x^2)}$$

$\log_a x$





1°

$a > 0$

$x+1 > 0$

$x > -1$

$x \in (-1, \infty)$

2°

$a \neq 1$

$x+1 \neq 1$

$x \neq 0$

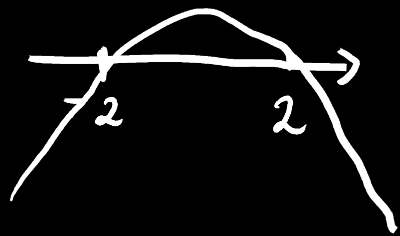
3°

$4 - x^2 > 0$

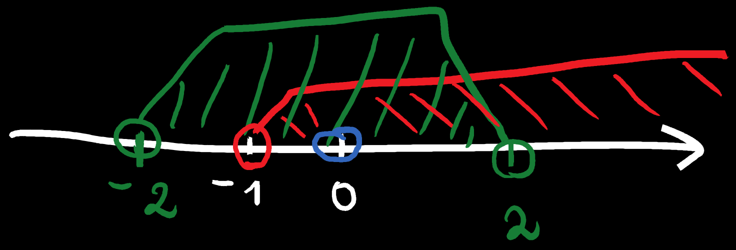
$(2-x)(2+x) > 0$

$\Downarrow \qquad \Downarrow$

$x = 2 \qquad x = -2$

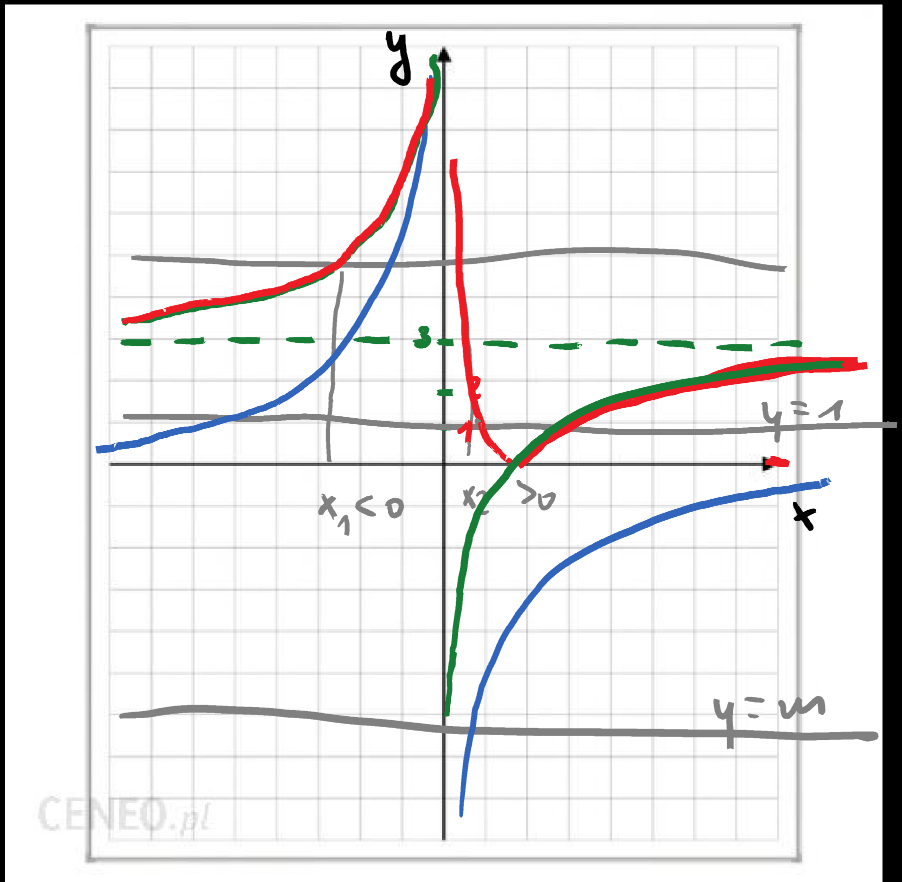


$x \in (-2, 2)$



$D_f = (-1, 2) \setminus \{0\}$

$$\underbrace{\left| 3 - \frac{1}{x} \right|}_{f(x)} = \underbrace{m}_{g(x)}$$



$f(x) = \left| 3 - \frac{1}{x} \right|$ $\quad x_1, x_2 > 0,\ x_1 \neq x_2$

$g(x) = m$

$y = \frac{-1}{x}$ $\quad$ II i IV

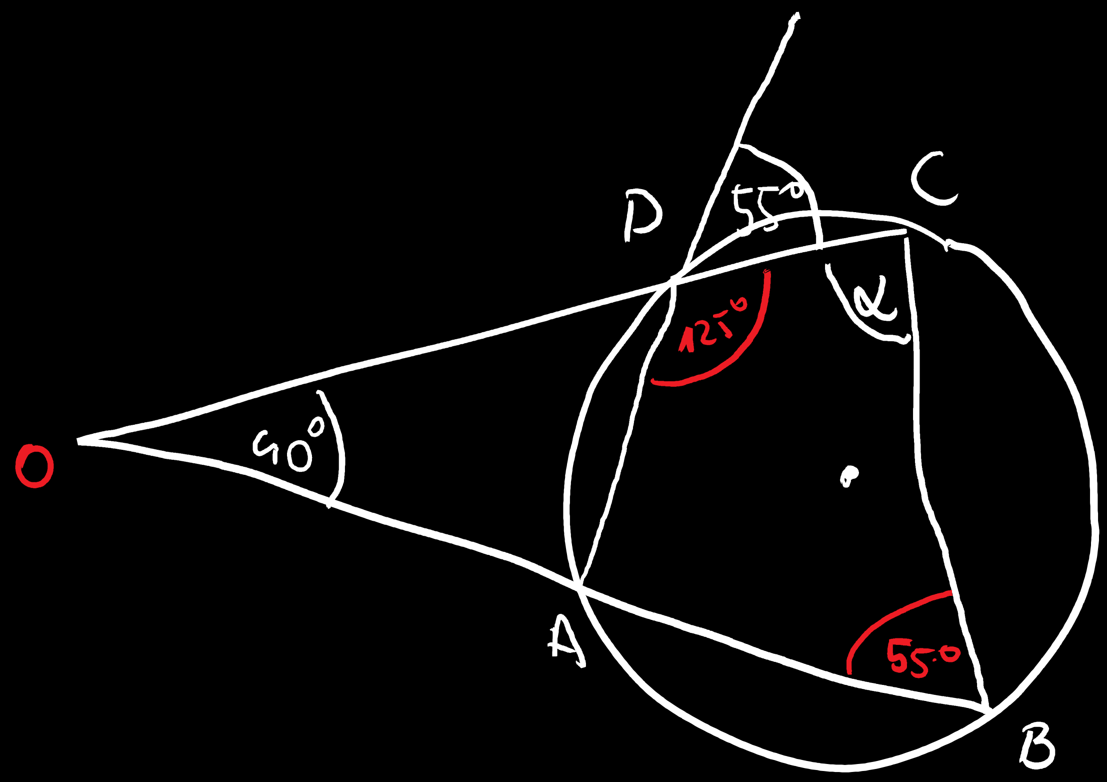
$y = \frac{a}{x}$ $\quad a < 0$

$y = \frac{-1}{x} + 3$

$m = 0$

$y = 0$

$m \in (0, 3)$



$\alpha = ?$

$180° - 55° = 125°$

$180° - 125° = 55°$

$40° + 55° + \alpha = 180°$

$\alpha = 85°$

4. $$f(x) = \frac{x+3}{(x-2)^2}$$

$$f'(x) = 0$$

$$f(x) = \frac{u}{v}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{(x+3)' \cdot (x-2)^2 - (x+3) \cdot [(x-2)^2]'}{(x-2)^4}$$

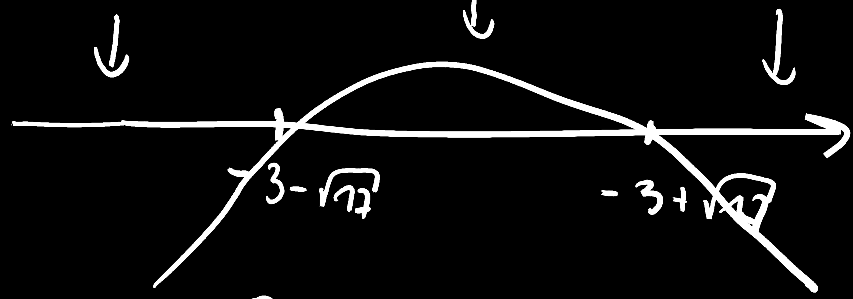
$$f'(x) = \frac{-x^2 - 6x + 8}{(x-2)^4}$$

$$f'(x) = \frac{1(x^2-4x+4) - (x+3)(x^2-4x+4)'}{(x-2)^2} = \frac{x^2-4x+4 - (x+3)(2x-4)}{(x-2)^2} =$$

$$= \frac{x^2-4x+4 - 2x^2 + 4x - 6x + 12}{(x-2)^2} = \frac{-x^2 - 6x + 16}{(x-2)^2}$$

$$f'(x) = 0$$

f' $\quad D_f = \mathbb{R} \setminus \{2\}$



$$\frac{-x^2 - 6x + 8}{(x-2)^4} = 0$$

$$-x^2 - 6x + 8 = 0$$

$$\Delta = 36 - 4 \cdot (-1) \cdot 8$$

$$\Delta = 36 + 32 = 68$$

$$\sqrt{\Delta} = \sqrt{4 \cdot 17} = 2\sqrt{17}$$

$$x_1 = \frac{6 - 2\sqrt{17}}{-2} = \frac{-2(-3+\sqrt{17})}{-2} = -3+\sqrt{17}$$

$$x_2 = \frac{6 + 2\sqrt{17}}{-2} = \frac{-2(-3-\sqrt{17})}{-2} = -3-\sqrt{17}$$

x	$(-\infty; -3-\sqrt{17})$	$-3-\sqrt{17}$	$(-3-\sqrt{17}, -3+\sqrt{17})$	$-3+\sqrt{17}$	$(-3+\sqrt{17}, \infty)$
f'	$-$		$+$		$-$
f	↘	min	↗	max	↘

1.

$a_1 = 16$

$a_3 = 1$

monotoniczny

$\nearrow$

$\searrow$

$\rightarrow$

$a_1 \cdot q = a_2$

$a_2 \cdot q = a_3$

$a_1 \cdot q^2 = a_3$

$16 \cdot q^2 = 1 \quad /:16$

$q^2 = \frac{1}{16} \quad /\sqrt{\ }$

$q = \frac{1}{4} \qquad q = -\frac{1}{4}$

niemonotoniczny

$S_n = ?$

ciąg geom

$S_n = a_1 \frac{1-q^n}{1-q}$

dla skończonego

dla nieskończ

$n \rightarrow \infty$

$q^n \rightarrow 0 \qquad q \in (-1, 1)$

$S_n = \lim_{n \to \infty} a_1 \frac{1-q^n}{1-q} = a_1 \frac{1}{1-q} = \frac{a_1}{1-q}$

$S_n = \frac{16}{1-(-\frac{1}{4})} = \frac{16}{1\frac{1}{4}} = 16 \cdot \frac{4}{5} = \frac{64}{5} = 12\frac{4}{5}$

6. $\lim\limits_{x \to 2} \left(\frac{x-3}{x+2} - \frac{x^3-52}{x^3+8} \right) = \frac{2-3}{2+2} - \frac{2^3-52}{2^3+8} =$

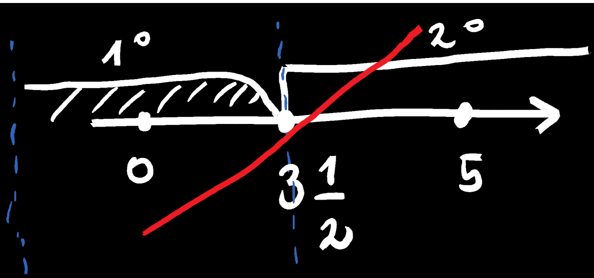
$= \frac{-1}{4} - \frac{8-52}{8+8} = -\frac{1}{4} - \frac{-44}{16} = -\frac{1}{4} + \frac{11}{4} = \frac{10}{4} = \frac{5}{2} = 2,5$

$\boxed{2 \mid 5 \mid 0}$

7.

$a>0$

$$3x - |2x - 7| < 11$$



I. $2x - 7 = 0$

$2x = 7$

$x = 3\frac{1}{2}$

1° $x \in \left(-\infty, 3\frac{1}{2}\right)$

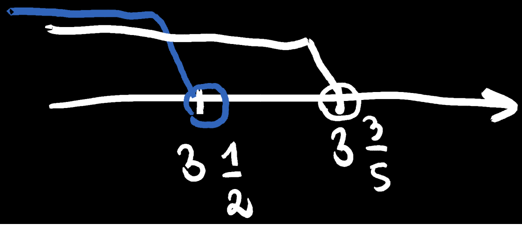
$3x + (2x - 7) < 11$

$3x + 2x - 7 < 11$

$5x < 18 \quad / : 5$

$x < \frac{18}{5}$

$x < 3\frac{3}{5}$



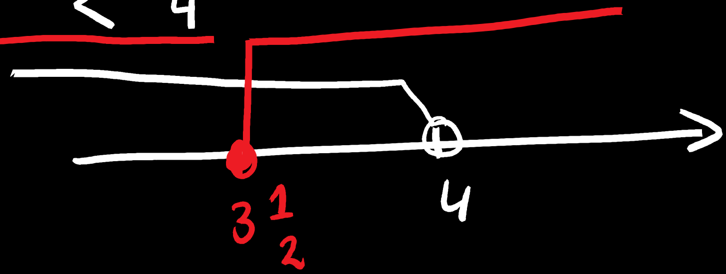
$x \in \left(-\infty, 3\frac{1}{2}\right)$

2° $x \in \left\langle 3\frac{1}{2}, \infty\right)$

$3x - (2x - 7) < 11$

$3x - 2x + 7 < 11$

$x < 4$



$x \in \left\langle 3\frac{1}{2}, 4\right)$

Rozwiązanie: $x \in (-\infty, 4)$

8.

$$\underbrace{\sin\left(\overset{\alpha}{x}+\overset{\beta}{\frac{\pi}{6}}\right)}+\cos x=\frac{3}{2} \qquad x\in\langle 0,2\pi\rangle$$

$$\left(\sin x\cdot\cos\frac{\pi}{6}+\cos x\cdot\sin\frac{\pi}{6}\right)+\cos x=\frac{3}{2}$$

$$\underbrace{(\sin x)\cdot\frac{\sqrt{3}}{2}}+(\cos x)\cdot\frac{1}{2}+\frac{2}{2}\cos x=\frac{3}{2}\ \Big/\cdot 2$$

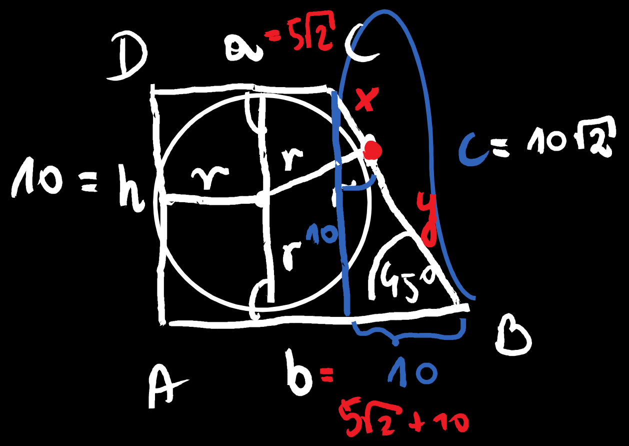
$$\sqrt{3}\sin x+\cos x+2\cos x=3$$

$$\sqrt{3}\sin x+3\cos x=3\ \Big/:3$$

$$\underset{\mathrm{tg}}{\frac{\sqrt{3}}{3}}\sin x+\cos x=1$$

$$\frac{\sqrt{3}}{3}\sin x + \cos x = 1$$

9.



$r = 5$

I

$10^2 + 10^2 = c^2$

$200 = c^2$

$\sqrt{2 \cdot 100} = c$

$10\sqrt{2} = c$

$a + b = h + x + y$

$a + b = 10 + (x + y)$

$a + b = 10 + c$

$a + b = 10 + 10\sqrt{2}$

$a + a + 10 = 10 + 10\sqrt{2}$

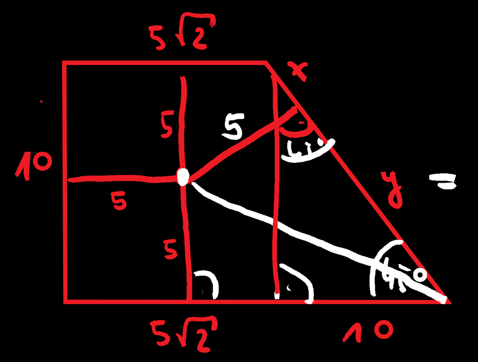
$2a = 10\sqrt{2} \quad / : 2$

$a = 5\sqrt{2}$

$b = (a + 10)$

$b = 5\sqrt{2} + 10$

$x = ?$ $\quad y = ?$



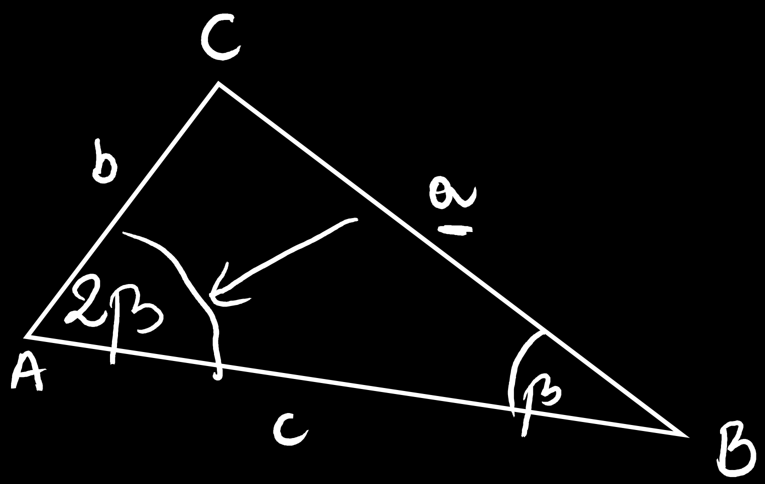
$y = 10$

II

$x = 10\sqrt{2} - 10$

$x = 10(\sqrt{2} - 1)$

(?) $\alpha = 2\beta \Rightarrow a^2 - b^2 = bc$



$$\sin 2\beta = 2\sin\beta \cdot \cos\beta$$

$$a^2 = b^2 + c^2 - 2 \cdot bc \cdot \cos 2\beta$$

$$- \quad b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$$

$$a^2 - b^2 = b^2 - a^2 -$$

$$\frac{a}{\sin 2\beta} = \frac{b}{\sin\beta}$$

$$\frac{a}{2\sin\beta\cos\beta} = \frac{b}{\sin\beta} \quad / \cdot \sin\beta$$

$$\frac{a}{2\cos\beta} = b \quad / \cdot 2\cos\beta$$

$$a = b \cdot 2\cos\beta \quad / : b$$

$$\frac{a}{b} = 2\cos\beta$$

$$\cos\beta = \frac{a}{2b} \quad / (\ )^2$$

$$\cos^2\beta = \frac{a^2}{4b^2}$$

$$\sin^2\beta + \cos^2\beta = 1$$

$$a = 2b\cos\beta$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta = \cos^2\beta - (1 - \cos^2\beta) = 2\cos^2\beta - 1$$

$$a^2 = b^2 + c^2 - 2bc\cos 2\beta$$

$$-\quad b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$a^2 - b^2 = b^2 - a^2 - 2bc\cos 2\beta + 2ac\cos\beta$$

$$a^2 - b^2 = b^2 - (2b\cos\beta)^2 - 2bc\underbrace{\cos 2\beta} + 2(2b\cos\beta)\cdot c\cdot\cos\beta$$

$$a^2 - b^2 = b^2 - \cancel{4b^2\cos^2\beta} - 2bc(2\cos^2\beta - 1) + \cancel{4bc\underbrace{\cos\beta\cdot\cos\beta}}$$

$$a^2 - b^2 = b^2 - 4\underbrace{bc}\cos^2\beta + 2\underbrace{bc}$$

$$\cos^2\beta = \frac{a^2}{4b^2}$$

$$a^2 - b^2 = b^2 - \cancel{4}\cancel{b}c\cdot\frac{a^2}{\cancel{4}b^{\cancel{2}}} + 2bc$$

$$a^2 - b^2 = \underbrace{b^2 - \frac{ca^2}{b}}_{-bc} + 2\underbrace{bc}$$

11. $W(\underline{x}) = 2x^3 + ax^2 + bx + c$

$W(a) = r$

$W(-1) = 6$

$W(-1) = 2\cdot(-1)^3 + a\cdot(-1)^2 + b\cdot(-1) + c$

$W(-1) = -2 + a - b + c$

$6 = -2 + a - b + c$

1) $8 = a - b + c$

1) $x^2 + x - 6$

2) $\dfrac{x+1}{x-a}$ $r = 6$

$a = ?$ $b = ?$

$c = ?$

$x + 1$

$x - (-1)$

$\shortparallel$

a

1) $x^2 + x - 6 = (x+3)(x-2)$ $y = a(x - x_1)(x - x_2)$

$\Delta = 1^2 - 4\cdot 1\cdot(-6)$

$\Delta = 1 + 24 = 25$

$\sqrt{\Delta} = 5$

$x_1 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{-1-5}{2} = \dfrac{-6}{2} = -3$

$x_2 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$

11. c. d

$$W(x) = 2x^3 + ax^2 + bx + c$$

$r = 0$

$W(a) = r$

$W(-3) = 0$

$W(2) = 0$

$(x+3)(x-2)$

$a = -3 \quad \underline{x+3} \quad r = 0$

$a = 2 \quad x - 2 \quad r = 0$

$x - a$

$$W(-3) = 2\cdot(-3)^3 + a\cdot(-3)^2 + b\cdot(-3) + c$$

$$W(-3) = -54 + 9a - 3b + c \implies 2)\ 0 = -54 + 9a - 3b + c$$

$$W(2) = 2\cdot 2^3 + a\cdot 2^2 + b\cdot 2 + c$$

$$W(2) = 16 + 4a + 2b + c \implies 3)\ 0 = 16 + 4a + 2b + c$$

11 c. d

$$\begin{cases} 1)\quad 8 = a - b + c \\ 2)\quad 0 = -54 + 9a - 3b + c \\ 3)\quad 0 = 16 + 4a + 2b + c \end{cases} \Rightarrow \begin{cases} {}^{*}c = \underbrace{8 - a + b} \\ 0 = -54 + 9a - 3b + 8 - a + b \\ 0 = 16 + 4a + 2b + 8 - a + b \end{cases}$$

$$\begin{cases} {}^{*}46 = 8a - 2b \quad / \cdot 3 \\ -24 = 3a + 3b \quad / \cdot 2 \end{cases}$$

$$+\begin{cases} 138 = 24a - \cancel{6b} \\ -48 = 6a + \cancel{6b} \end{cases}$$

$$90 = 30a \quad /:30$$

$$\underline{3 = a}$$

$$46 = 8 \cdot 3 - 2b$$

$$22 = -2b \quad /:(-2)$$

$$\underline{-11 = b}$$

$$c = 8 - 3 - 11$$

$$\underline{c = -6}$$

12. Ze zbioru wszystkich liczb naturalnych dodatnich nie większych od 30 losujemy kolejno 2 razy po jednej liczbie bez zwracania. Oblicz prawdopodobieństwo tego, że otrzymamy w ten sposób parę liczb, których iloczyn jest mniejszy od 30 pod warunkiem, że pierwsza wylosowana liczba jest mniejsza od drugiej wylosowanej liczby.

$n \leqslant 30$ 2 razy po 1 bez zwracania

$A: x \cdot y < 30$

$B: x < y$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\overline{\overline{A\cap B}}}{\overline{\overline{B}}} = \frac{\cancel{50}^{10}}{\cancel{15}_{3} \cdot 29} = \frac{10}{87}$$

$\overline{\overline{\Omega}} = 30 \cdot 29$

$P(B) = \frac{\overline{\overline{B}}}{\overline{\overline{\Omega}}}$

$P(A\cap B) = \frac{\overline{\overline{A\cap B}}}{\overline{\overline{\Omega}}}$

$A\cap B = \{(3,4)(3,5)(3,6)(3,7)(3,8)(3,9),(4,5)(4,6)(4,7)\}$

$B = \{(\ ,1)\ (3,1)(3,2) \quad \}$

$2 \to 1,\ 3 \to 2,\ 4 \to 3,\ 5 \to 4,\ \ldots,\ 30 \to 29$

$\overline{\overline{B}} = 1 + 2 + 3 + 4 + \ldots + 29$

$a_1 = 1 \quad a_n = 29 \quad n = 29 \quad r = 1$

$\overline{\overline{B}} = S_{29} = \frac{a_1 + a_n}{2} \cdot n = \frac{1+29}{2} \cdot 29 = 15 \cdot 29$

435

$\overline{\overline{A\cap B}} = 29 + 12 + 6 + 3 = 50$

$A\cap B = \{(1,2)(1,3), \ldots (1,29)$

$(2,3)(2,4)(2,5) \ldots (2,14)\}$

13.

$$(m+1)x^2 + 2\sqrt{2}x - m^2 + 2 = 0$$

1) $x_1^2 + x_2^2 \geq m - x_1 x_2$

2) $x_1 \neq x_2$

$ax^2 + bx + c$

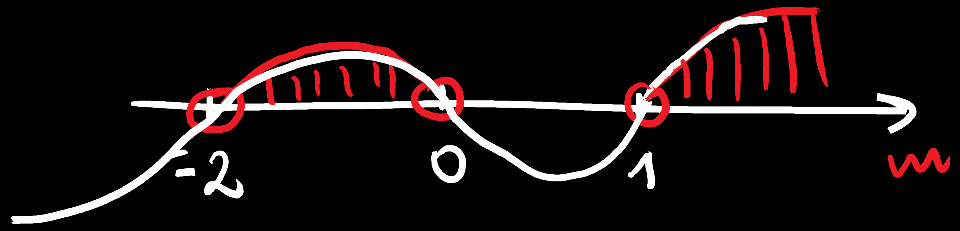
$\Delta > 0$

$m^3 + m^2 - 2m > 0$

$m(m^2 + m - 2) > 0$

$m^2 + m - 2 > 0$

$m = 0$ $m = 1$ $m = -2$



$m \in (-2, 0) \cup (1, \infty)$

13.c.d

1) $x_1^2+x_2^2 \geq m \cdot \underbrace{x_1 \cdot x_2}$

$a = m+1$

$b = 2\sqrt{2}$

$c = -m^2+2$

$x_1 \cdot x_2 = \frac{c}{a}$

$x_1+x_2 = -\frac{b}{a}$

$\Big(\underbrace{x_1^2}_{a^2} + \underbrace{2x_1x_2}_{2ab} + \underbrace{x_2^2}_{b^2}\Big) - 2x_1x_2 - m + x_1x_2 \geq 0$

$(x_1+x_2)^2 - x_1x_2 - m \geq 0$

$\left(-\frac{b}{a}\right)^2 - \frac{c}{a} - m \geq 0$

$\left(-\frac{2\sqrt{2}}{(m+1)}\right)^2 - \frac{-m^2+2}{m+1} - m \geq 0$

13. c. d

1) $x_1^2 + x_2^2 \geq m \cdot \underbrace{x_1 \cdot x_2}$

$$\frac{8}{(m+1)^2} + \frac{(m^2-2)(m+1)}{(m+1)^2} - \frac{m(m+1)^2}{(m+1)^2} \geq 0$$

$$\frac{8 + m^3 + m^2 - 2m - 2 - m(m^2 + 2m + 1)}{(m+1)^2} \geq 0$$

$$\frac{6 + \cancel{m^3} + m^2 - 2m - \cancel{m^3} - 2m^2 - m}{(m+1)^2} \geq 0$$

$$\frac{-m^2 - 3m + 6}{(m+1)^2} \geq 0$$

$a = m+1$

$b = 2\sqrt{2}$

$c = -m^2 + 2$

$x_1 \cdot x_2 = \frac{c}{a}$

$x_1 + x_2 = -\frac{b}{a}$

$m \neq -1$

$-m^2 - 3m + 6 \geq 0$

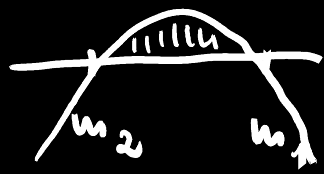
$\Delta = 9 + 4 \cdot 6 = 33$

$\sqrt{\Delta} = \sqrt{33}$

$m_1 = \frac{3 - \sqrt{33}}{-2}$

$m_2 = \frac{3 + \sqrt{33}}{-2}$

$m \in \left\langle \frac{-3-\sqrt{33}}{2}, \frac{-3+\sqrt{33}}{2} \right\rangle$



14. 1) $a + b + c + d = 272$

a, b, c, d - ciąg g.

2) $c = a + 48$

$a, b, c, d \in \mathbb{C}$

$a + b + a + 48 + d = 272$

$2a + b + d = 224$

3) $b^2 = a \cdot c$ 4) $c^2 = b \cdot d$

---

1) $a + aq + aq^2 + aq^3 = 272$

2) $aq^2 = a + 48$

1) $a + aq + a + 48 + aq^3 = 272$

$2a + aq + aq^3 = 224$

$a \cdot (2 + q + q^3) = 224$

$a = \frac{224}{2 + q + q^3}$

$\frac{224q^2}{2 + q + q^3} = \frac{224}{2 + q + q^3} + 48 \quad / \cdot (2 + q + q^3)$

$224q^2 = 224 + 48(2 + q + q^3)$

$224q^2 = 224 + 96 + 48q + 48q^3$

$0 = 48q^3 - 224q^2 + 48q + 320 \quad / : 8$

$0 = 6q^3 - 28q^2 + 6q + 40 \quad / : 2$

$$0 = \boxed{3}q^3 - 14q^2 + 6q + 20$$

$$p = \{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20\}$$

$$q = \{\pm 1, \pm 3\}$$

$$\frac{p}{q} = \{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{20}{3}\}$$

$W(1) = 3 - 14 + 6 + 20 \neq 0$

$W(-1) = -3 - 14 - 6 + 20 \neq 0$

$W(2) = 24 - 56 + 12 + 20 = 0$

$a = 2$

$q - a \qquad (q - 2)$

$$\begin{array}{l} 3q^2 - 8q - 10 \\ \hline (3q^3 - 14q^2 + 6q + 20) : (q - 2) \\ -3q^3 + 6q^2 \\ \hline = \quad -8q^2 + 6q + 20 \\ \quad\;\; 8q^2 - 16q \\ \hline = \quad\quad -10q + 20 \\ \quad\quad\;\; 10q - 20 \\ \hline \quad\quad = \quad = \end{array}$$

$$0 = (q-2)\cdot(3q^2 - 8q - 10)$$

$$\Downarrow \qquad \Downarrow$$

$$\boxed{q = 2} \in \mathbb{C} \qquad 3q^2 - 8q - 10 = 0$$

$\Delta = 64 - 4 \cdot 3 \cdot (-10) = 64 + 120$

$\Delta = 184 \qquad \sqrt{\Delta} = \sqrt{4 \cdot 46} = 2\sqrt{46}$

$q_1, q_2 \notin \mathbb{C}$

$W(4) = 192 - 224 + 24 + 20 \neq 0$

$$a = \frac{224}{2+q+q^3}$$

$q = 2 \in \mathbb{C}$

$$a = \frac{224}{2+2+2^3} = \frac{224}{12} =$$

15. $f(x) = \frac{1}{4}x^2 - 1$

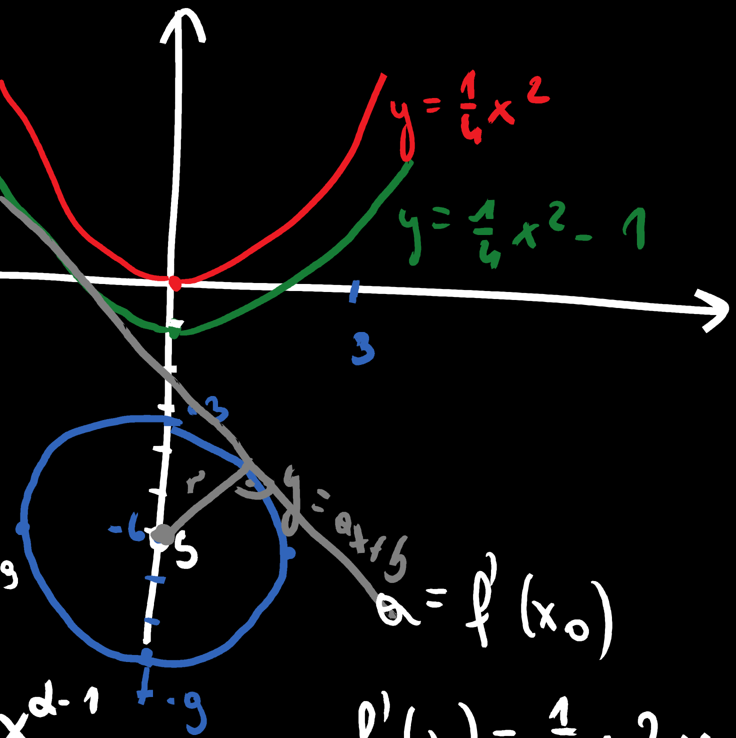
$f(x_0) = \frac{1}{4}x_0^2 - 1$

$x^2 + (y+6)^2 = 8$

$(x-0)^2$

$(x-a)^2 + (y-b)^2 = r^2$ $\qquad S(a,b)$

$S(0,-6) \qquad r = 2\sqrt{2}$



$y = ax^2$

$W(0,0)$

$P\left(x_0, \frac{1}{4}x_0^2 - 1\right)$

$\boxed{y = ax + b}$

$a = f'(x_0)$

$b = f(x_0) - f'(x_0)\cdot x_0$

$c' = 0$

$(x^{50})' = 50x^{49}$

$(x^{\alpha})' = \alpha x^{\alpha-1}$

$(x^3)' = 3x^2$

$a = f'(x_0)$

$f'(x) = \frac{1}{4}\cdot 2x = \frac{1}{2}x$

$a = f'(x_0) = \frac{1}{2}x_0$

$b = \frac{1}{4}x_0^2 - 1 - \frac{1}{2}x_0\cdot x_0$

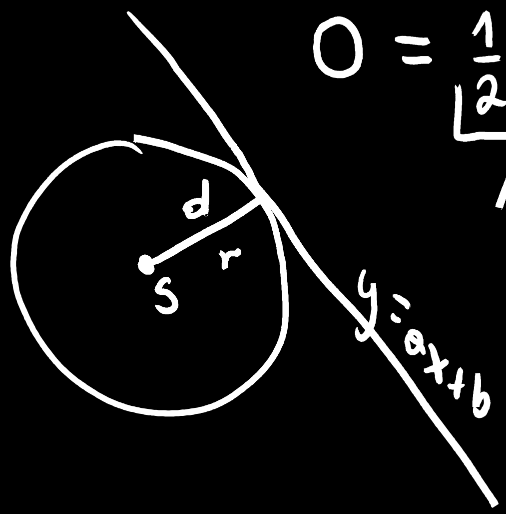
$b = \frac{1}{4}x_0^2 - 1 - \frac{2}{4}x_0^2 = \boxed{-\frac{1}{4}x_0^2 - 1}$

$y = ax + b$ $\qquad a = \frac{1}{2}x_0$ $\qquad b = -\frac{1}{4}x_0^2 - 1$ $\qquad S(\overset{x_0}{0}, \overset{y_0}{-6})$

$r = 2\sqrt{2}$

$$\boxed{y = \frac{1}{2}x_0 X - \frac{1}{4}x_0^2 - 1}$$

$$\boxed{d = r}$$



$$0 = \underbrace{\frac{1}{2}x_0}_{A} X \underbrace{-}_{B} y \underbrace{-\frac{1}{4}x_0^2 - 1}_{C}$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$\frac{\left|\overbrace{\frac{1}{2}x_0}^{A} \cdot 0 - 1 \cdot (-6) - \frac{1}{4}x_0^2 - 1\right|}{\sqrt{\frac{1}{4}x_0^2 + 1}} = 2\sqrt{2}$$

$$\frac{\left|5 - \frac{1}{4}x_0^2\right|}{\sqrt{\frac{1}{4}x_0^2 + 1}} = 2\sqrt{2}$$

$y = ax + b$

$\underbrace{A}x + \underbrace{B}y + \underbrace{C} = 0$

$$\left|5-\frac{1}{4}x_0^2\right| = 2\sqrt{2}\sqrt{\frac{1}{4}x_0^2+1} \qquad /(\ )^2$$

$a^2 - b^2$

$$\left(5-\frac{1}{4}x_0^2\right)^2 = 8\left(\frac{1}{4}x_0^2+1\right)$$

$$25-\frac{5}{2}x_0^2+\frac{1}{16}x_0^4 = 2x_0^2+8 \quad /\cdot 16$$

$$400-40x_0^2+x_0^4 = 32x_0^2+128$$

$$x_0^4-72x_0^2+272 = 0$$

$$t = x_0^2 \quad , \quad t \geq 0$$

$$t^2-72t+272 = 0$$

<0

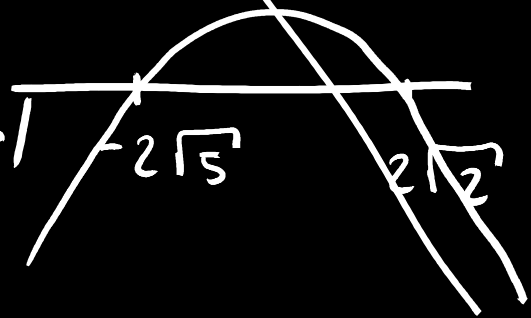
$$\left(\sqrt{5}-\frac{1}{2}x_0\right)\left(\sqrt{5}+\frac{1}{2}x_0\right) > 0$$

$\Downarrow$

$\sqrt{5} = \frac{1}{2}x_0$ $\qquad x_0 = -2\sqrt{5}$

$2\sqrt{5} = x_0$

$\sqrt{a}^2 = |a|$



$5 \cdot 2 \cdot \frac{1}{4}$

$$t^2 - 72t + 272 = 0$$

$$\Delta = 5184 - 1088 = 4096 \qquad \sqrt{\Delta} = 64$$

$$t_1 = \frac{72 - 64}{2} = 4$$

$$t = x_0^2$$

$$t_2 = \frac{72 + 64}{2} = \frac{136}{2} = 68$$

$$4 = x_0^2 \qquad 68 = x_0^2$$

$$\boxed{y = \frac{1}{2}x_0 X - \frac{1}{4}x_0^2 - 1}$$

1) $2 = x_0$

2) $-2 = x_0$

3) $2\sqrt{17} = x_0$

4) $-2\sqrt{17} = x_0$

1) $y = x - 2$

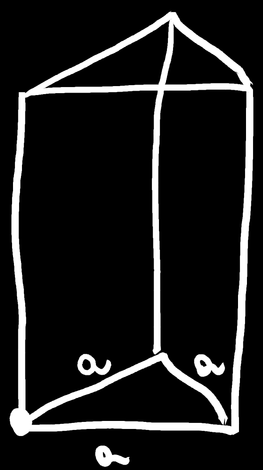
2) $y = -x - 2$

3) $y = \sqrt{17}\,x - 18$

4) $y = -\sqrt{17}\,x - 18$

W graniastosłupie prawidłowym trójkątnym suma długości trzech różnych krawędzi wychodzących z jednego wierzchołka wynosi S. Wyznacz objętość tego graniastosłupa jako funkcję długości jednej z jego krawędzi i podaj dziedzinę tej funkcji. Oblicz wymiary graniastosłupa, którego objętość jest największa. Oblicz tę objętość.

$S-2a=H$



$$H + 2a = S \implies H = S - 2a$$

$$V = ?$$

$$V = P_p \cdot H = \frac{a^2\sqrt{3}}{4} \cdot H = \frac{a^2\sqrt{3}}{4} \cdot (S-2a)$$

$$V(a) = \frac{a^2\sqrt{3}(S-2a)}{4}$$

$V(a)$ najw.

$$V(a) = \frac{Sa^2\sqrt{3}}{4} - \frac{2a^3\sqrt{3}}{4}$$

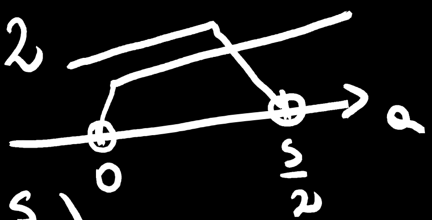
D: $H > 0 \quad a > 0$

$S - 2a > 0$

$S > 2a \; / : 2$

$\frac{S}{2} > a$

$a \in \left(0, \frac{S}{2}\right)$



$$V(a) = \frac{s\sqrt{3}\,a^2}{4} - \frac{2\sqrt{3}\,a^3}{4}$$

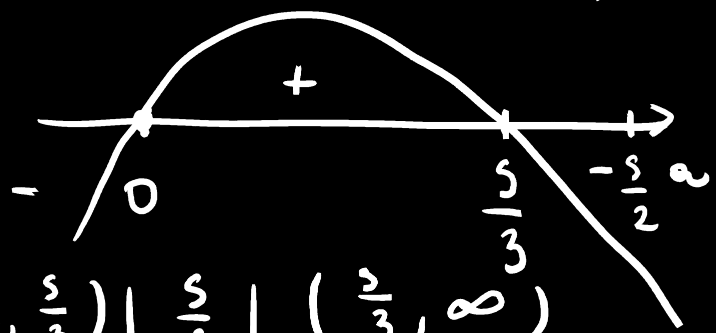
$$\boxed{a \in \left(0, \frac{s}{2}\right)}$$

$$\boxed{V'(a) = 0}$$

$$(x^{\alpha})' = \alpha x^{\alpha - 1}$$

$$V'(a) = \frac{s\sqrt{3}}{\cancel{4}\,2}\cancel{2}a - \frac{\cancel{2}\sqrt{3}}{\cancel{4}\,2}3a^2 = \frac{s\sqrt{3}}{2}a - \frac{3\sqrt{3}}{2}a^2$$

wykres $f'(a)$



$$\frac{s\sqrt{3}}{2}a - \frac{3\sqrt{3}}{2}a^2 = 0 \quad / \cdot \frac{2}{\sqrt{3}}$$

$$\boxed{sa - 3a^2 = 0}$$

$$a \cdot (s - 3a) = 0$$

$\Downarrow$ $\Downarrow$

$\boxed{a = 0}$ $\quad s - 3a = 0$

$s = 3a$

$\boxed{a = \frac{s}{3}}$

a	$(-\infty, 0)$	0	$(0, \frac{s}{3})$	$\frac{s}{3}$	$(\frac{s}{3}, \infty)$
$f'(a)$	$-$	0	$+$	0	$-$
$f(a)$	$\searrow$	min	$\nearrow$	$\frac{\sqrt{3}s^3}{108}$ max	$\searrow$

$$V(a) = \frac{S\sqrt{3}\, a^2}{4} - \frac{2\sqrt{3}\, a^3}{4}$$

$$V_{najw} = V\left(\frac{S}{3}\right) = \frac{S\sqrt{3} \cdot \left(\frac{S}{3}\right)^2}{4} - \frac{2\sqrt{3} \cdot \left(\frac{S}{3}\right)^3}{4} =$$

$$= \frac{S\sqrt{3} \cdot \frac{S^2 \cdot 3}{9 \cdot 3} - 2\sqrt{3} \cdot \frac{S^3}{27}}{4} = \frac{3\sqrt{3}\, S^3 - 2\sqrt{3}\, S^3}{27 \cdot 4} =$$

$$= \frac{\sqrt{3}\, S^3}{108}$$