1.Dany jest ułamek dziesiętny nieskończony okresowy 0,1(2345). Na setnym miejscu po przecinku znajduje się w nim cyfra A. 2. B. 3. C. 4. D. 5.

4 cyfry

$$0,1(2345) = 0,1\,2345\,2345\,2345$$

$$0,1\,\underbrace{2345}_{\times 24}$$

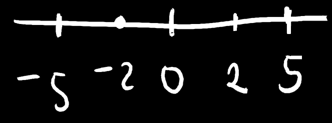
24

| 5 | 2 | 3 | 4 |
|---|---|---|---|

96 97 98 99 100

$$\frac{(-2)^3 \cdot \left(\frac{1}{2}\right)^{-5}}{\left(\sqrt{8}\right)^6} = \frac{-2^3 \cdot 2^5}{\left[\left(2^3\right)^{\frac{1}{2}}\right]^6} = -\frac{2^8}{2^9} =$$

$$= -2^{-1} = -\frac{1}{2}$$




$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\left(a^n\right)^m = a^{n \cdot m}$$

liczba przeciwna : $\frac{1}{2}$

3. Cenę pewnego towaru podwyższono o 20%, następnie otrzymaną w ten sposób nową cenę obniżono o 20%. Cena końcowa jest A. o 4% wyższa od ceny początkowej. C. o 4% niższa od ceny początkowej. B. o 2% niższa od ceny początkowej. D. równa cenie początkowej.

1x - cena początkowa

$x + 20\% x = 1x + 0{,}2x = \underbrace{1{,}2x}$ - cena po podwyżce

$1{,}2x - 20\% \cdot 1{,}2x = 1{,}2x - 0{,}2 \cdot 1{,}2x = 1{,}2x - 0{,}24x =$

$= 0{,}96x$ - cena po obniżce

$1x - 0{,}96x = 0{,}04x = 4\% x$

$$\frac{1}{\sqrt{3}-2}-\frac{1}{\sqrt{3}+2}=\frac{1\,(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)}-\frac{1\,(\sqrt{3}-2)}{(\sqrt{3}+2)(\sqrt{3}-2)}=$$

$$(a-b)\cdot(a+b)=a^2-b^2$$

$$=\frac{\sqrt{3}+2}{\sqrt{3}^2-2^2}-\frac{\sqrt{3}-2}{\sqrt{3}^2-2^2}=$$

$$=\frac{\sqrt{3}+2}{3-4}-\frac{\sqrt{3}-2}{3-4}=$$

$$=\frac{\sqrt{3}+2}{-1}-\frac{\sqrt{3}-2}{-1}=$$

$$=-\cancel{\sqrt{3}}-2+\cancel{\sqrt{3}}-2=-4$$

5. Kwotę 5000 zł ulokowano w banku na lokacie oprocentowanej 3% w stosunku rocznym, z odsetkami kapitalizowanymi co rok. Przy każdej kapitalizacji od odsetek pobiera się podatek w wysokości 19%. Kwota lokaty po dwóch latach wyniesie:

$K_0 = 5000$ zł

$p = 3\%$

$3\% \cdot 81\%$

$0{,}03 \cdot 0{,}81$

po I roku

$3\% \cdot 5000 =$

$= 0{,}03 \cdot 5000 =$

$= 150$

$81\% \cdot 150 =$

$= 0{,}81 \cdot 150 =$

$= 121{,}50$

po I roku $\underline{5121{,}50}$

$$K_2 = \underset{5000}{K_0}\left(1 + \frac{\overset{3\%}{p}}{100}\right)^{\overset{2}{n}}$$

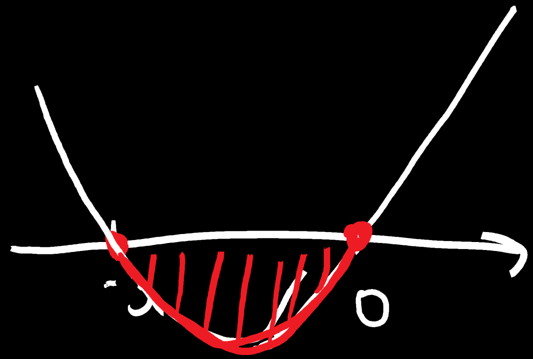
$$K_2 = 5000\left(1 + 0{,}03 \cdot 0{,}81\right)^2$$

$$2x \cdot (x+3) \leq 0$$

$\Downarrow$ $\Downarrow$

$2x = 0 \quad / :2$ $\qquad x+3=0$

$x = 0$ $\qquad x = -3$




$x \in \langle -3, 0 \rangle$

$$\begin{cases} 2x - 3y = -1 & / \cdot 3 \\ -6x + \underline{a}y = 3 \end{cases}$$

$$+\begin{cases} 6x - 9y = -3 \\ -6x + ay = 3 \end{cases}$$

$$= -9y + ay = 0$$

$$(-9 + a)y = 0$$

$$\Downarrow$$

$$-9 + a = 0$$

$$\boxed{a = 9}$$

$$0 = 0$$

$$-$$

$$\begin{cases} y = 0 \\ x = \end{cases}$$

8.

$$(x^2-1)\cdot(x^2+5x)=0$$

$\Downarrow$ $\Downarrow$

$x^2-1=0$

$a^2-b^2=(a-b)(a+b)$

$(x-1)(x+1)=0$

$\Downarrow$ $\Downarrow$

$x=1$ $x=-1$

$x^2+5x=0$

$x\cdot(x+5)=0$

$\Downarrow$ $\Downarrow$

$x=0$ $x=-5$

4 rozw

suma $=-5$

Liczba a = 2,2 jest przybliżeniem z nadmiarem liczby x. Błąd bezwzględny tego przybliżenia jest równy 0,004, gdy A. x = 2,204.
B. x = 2,24 C. x = 2,16 D. x = 2,196.

przybliż

l. rzeczyw.

bł bezwzgl = | w. rzeczyw. − w. przybliż |

A. bł bezwzgl = $|2{,}204 - 2{,}2| = 0{,}004$

B. bł bezwzgl = $|2{,}24 - 2{,}2| = 0{,}04$

C. bł bezwzgl = $|2{,}16 - 2{,}2| = |-0{,}04| = 0{,}04$

D. bł bezwzgl = $|2{,}196 - 2{,}2| = |-0{,}004| = 0{,}004$

$\underbrace{\log 3} = a$

A) $\log \frac{100}{27} = \frac{2}{a^3}$ B) $\log \frac{100}{27} = \frac{2}{3a}$ C) $\log \frac{100}{27} = 3a - 2$ D) $\log \frac{100}{27} =$

$= 2 - 3a$

$\log \frac{100}{27} = \log 100 - \log 27 = 2 - \log 3^3 = 2 - 3\underbrace{\log 3} =$

$= 2 - 3a$

$\log_a b - \log_a c = \underbrace{\log_a \frac{b}{c}}$

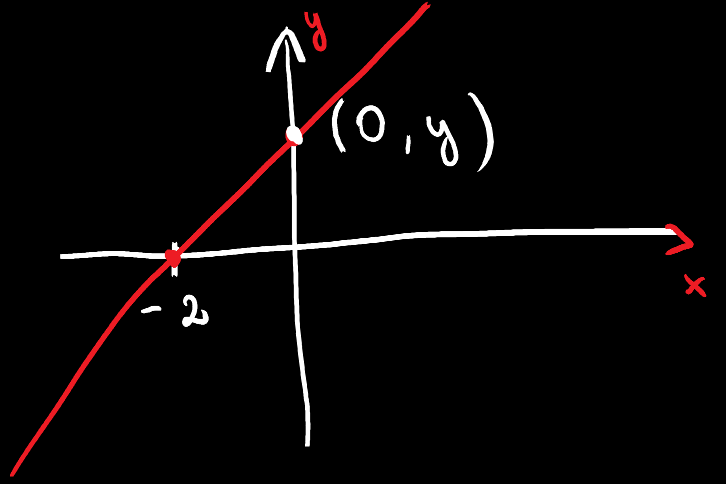
$\log_a b^n = n \log_a b$

Funkcja liniowa f określona wzorem f(x)= 2x +b osiąga wartości dodatnie tylko wtedy, gdy x >-2. Punkt przecięcia wykresu funkcji f z osią OY to

$f(x) = 2x + b$

$f(x) > 0$ dla $x > -2$

OY : $(0, 4)$




$(-2, 0)$

$0 = 2 \cdot (-2) + b$

$0 = -4 + b$

$4 = b$

$f(x) = 2x + 4$

$f(0) = 2 \cdot 0 + 4$

$f(0) = 4$

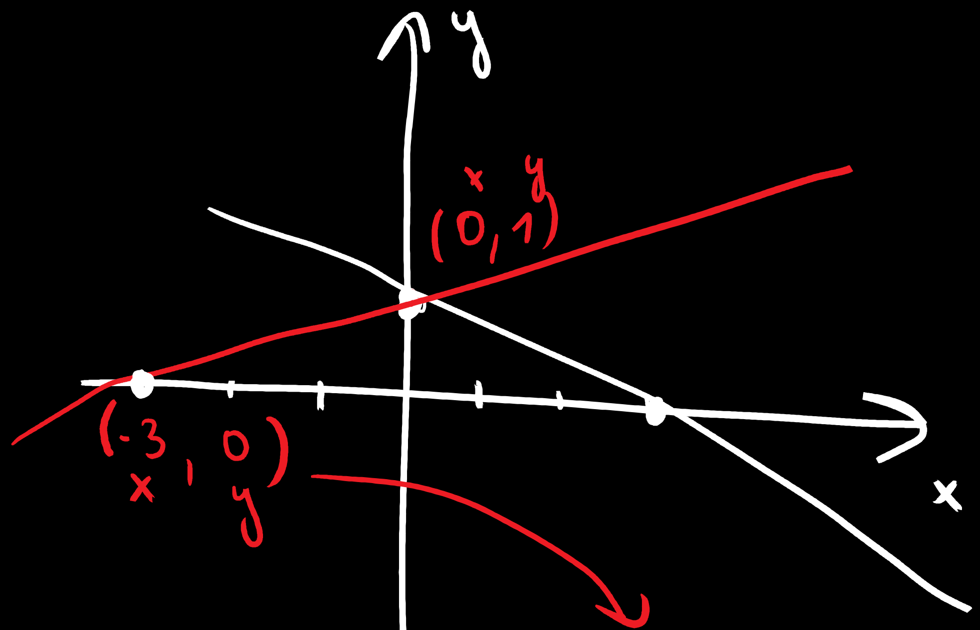
OY

$$f(x) = -\frac{1}{3}\underbrace{(x+1)}_{1 \leftarrow} + \underbrace{\frac{4}{3}}_{\frac{4}{3}\uparrow}$$

$$y = \underbrace{\underbrace{-\frac{1}{3}}\,\underbrace{x}}_{a = -\frac{1}{3}} \qquad y = \underline{a}\,x \quad (0,0)$$

$$f(x) = -\frac{1}{3}x - \frac{1}{3} + \frac{4}{3}$$

$$f(x) = -\frac{1}{3}x + 1$$




OY: $x = 0$, $y = 1$ $\quad (0,1)$

OX: $y = 0$ $\quad x = 3$

$0 = -\frac{1}{3}x + 1$ $\quad (3,0)$

$\frac{1}{3}x = 1 \;/\cdot 3$

$y = ax + b \longrightarrow y = ax + 1 \longrightarrow \boxed{y = \frac{1}{3}x + 1}$

$1 = a \cdot 0 + b$ $\qquad 0 = -3a + 1$

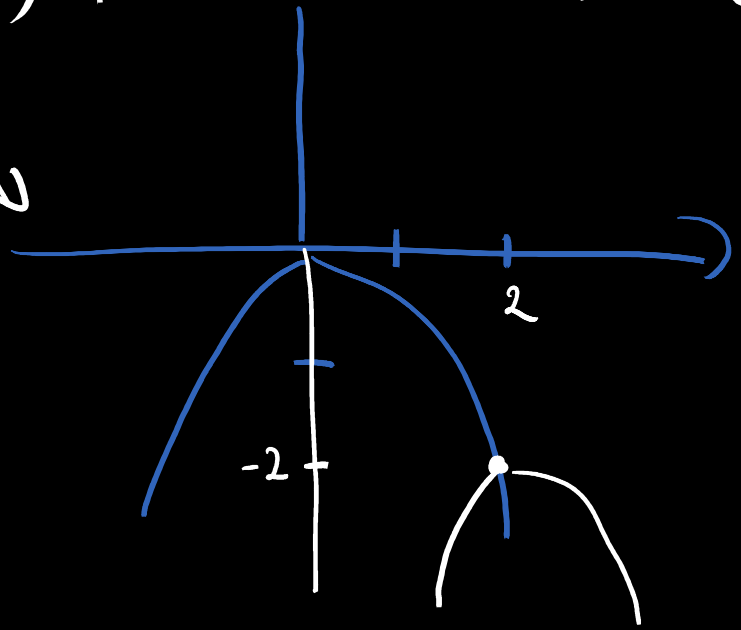
$b = 1$ $\qquad 3a = 1 \qquad a = \frac{1}{3}$

$$Z_w = (-\infty, -2\rangle$$

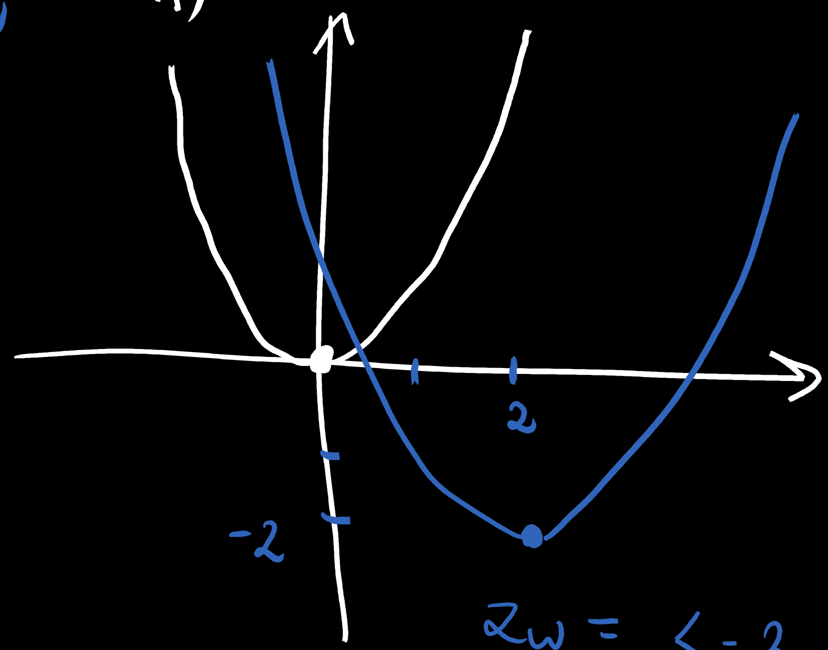
A) $f(x) = 3\underbrace{(x-2)}_{2 \rightarrow}{}^2 \underbrace{- 2}_{2\downarrow}$

$y = \underbrace{ax^2}$ $\qquad y = 3x^2$

B) $f(x) = -3(x-2)^2 - 2$

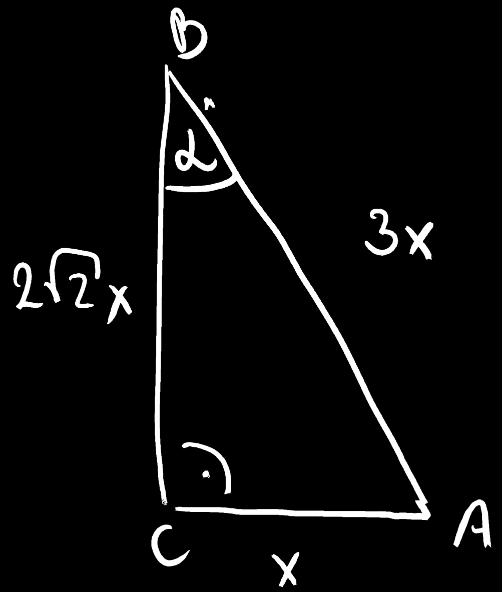



A)




$Z_w = \langle -2, \infty)$

W pewnym trójkącie prostokątnym przeciwprostokątna jest trzy razy dłuższa od jednej z przyprostokątnych. Wartość cosinusa mniejszego kąta ostrego tego trójkąta jest równa




$$a^2 + x^2 = (3x)^2$$

$$a^2 + x^2 = 9x^2$$

$$a^2 = 8x^2$$

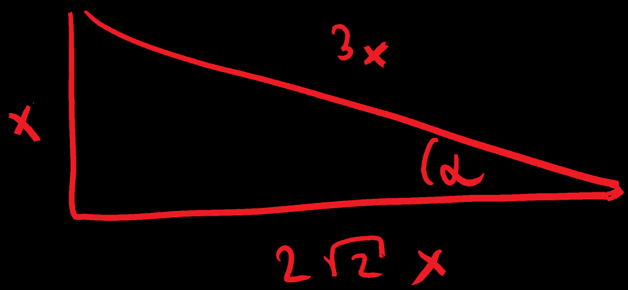
$$a = \sqrt{8}\,x$$

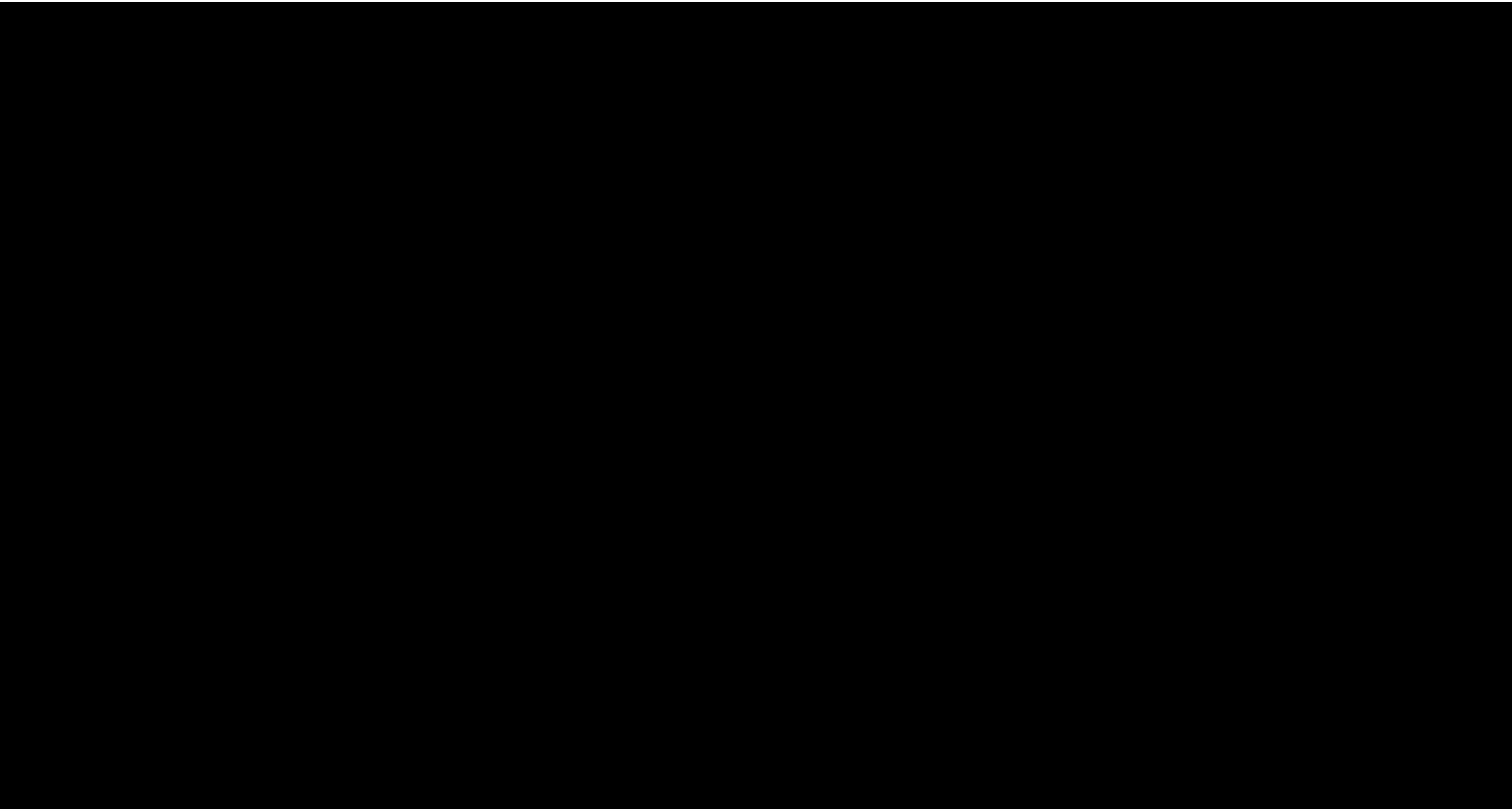
$$a = 2\sqrt{2}\,x$$

$$x,\ 2\sqrt{2}\,x,\ 3x$$

$$\cos\alpha = \frac{2\sqrt{2}\,x}{3x}$$

$$\cos\alpha = \frac{2\sqrt{2}}{3}$$





$\underline{a_n} = (5-n)(n+3)$

$\underbrace{n \geq 1}_{n \in \mathbb{N}}$

(?) ile jest $a_n > 0$

$a_n > 0$

$\uparrow$

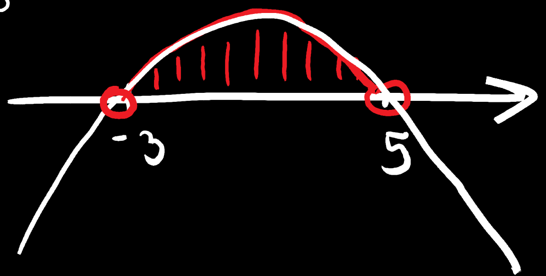
$(5-n)\cdot(n+3) \underbrace{>}\ 0$

$\Downarrow$ $\Downarrow$

$5-n=0$ $n+3=0$

$n=5$ $n=-3$

$a<0$




$n \in (-3, 5)$

$n \in \{1, 2, 3, 4\}$

4

$\underbrace{1}_{a_{n-1}}, \underbrace{a+1}_{a_n}, \underbrace{9}_{a_{n+1}}$ – ciąg geometryczny

$a_n^2 = a_{n-1} \cdot a_{n+1}$

$n-1, n, n+1$

$2, 3, 4$

$(a+1)^2 = 1 \cdot 9$

$a^2 + 2a + 1 = 9$

$a^2 + 2a - 8 = 0$

$\Delta = b^2 - 4ac$

$\Delta = 4 - 4 \cdot 1 (-8)$

$\Delta = 4 + 32 = 36$

$\sqrt{\Delta} = 6$

$a_1 = \frac{-2-6}{2} = \frac{-8}{2} = -4$

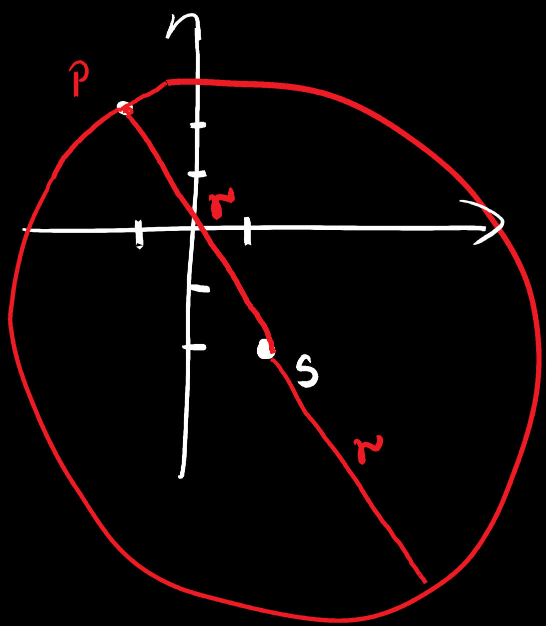
$a_2 = \frac{-2+6}{2} = \frac{4}{2} = 2$

D

$S(1, -2)$ $\quad x_S \quad y_S$

$P(-1, 2)$ $\quad x_P \quad y_P$

średnica = ?




$d = 2r$

$r = |PS|$

$|PS| = \sqrt{(x_S - x_P)^2 + (y_S - y_P)^2}$

$|PS| = \sqrt{(1+1)^2 + (-2-2)^2}$

$|PS| = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$d = 4\sqrt{5}$

A

$k: y = \underbrace{-3}_{a}x + 4$

$k \perp l$

$a_1 \cdot a_2 = -1$

$-3 \cdot a = -1 \quad / : (-3)$

$a = \frac{1}{3}$

A

$l: y = \underbrace{a}_{a_2}x + b$

$k \perp l$

$l: y = \frac{1}{3}x + b$

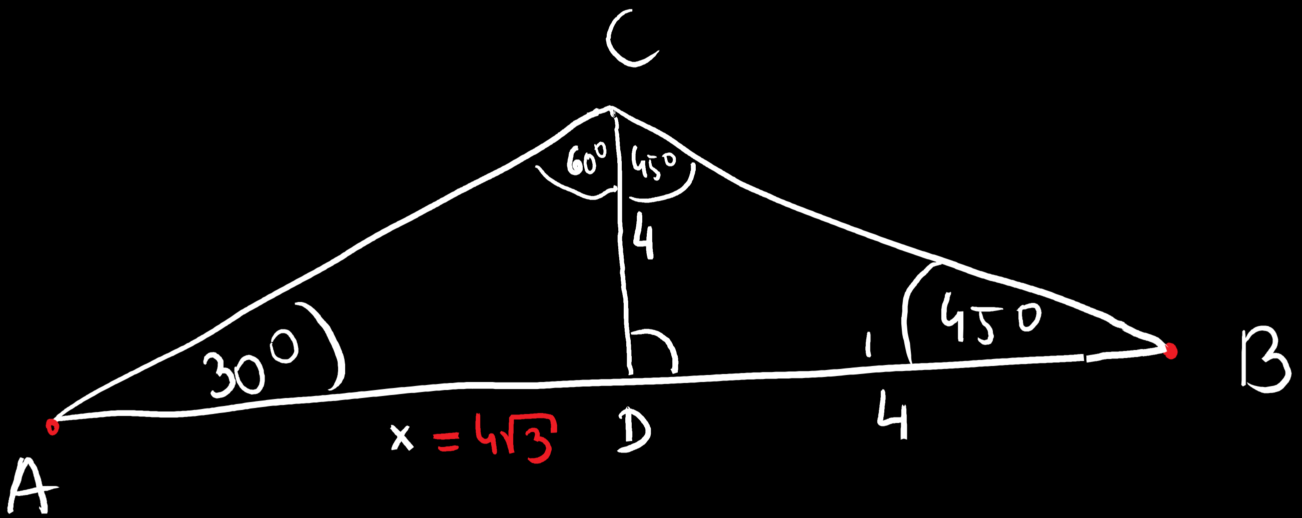
$0 = \frac{1}{3}x - y + b \quad / \cdot 3$

$0 = x - 3y + 3b$

$30°, 45°, 105°$

$h = 4$

$|AB| = ?$




$|AB| = 4\sqrt{3} + 4$

$|AB| = 4(\sqrt{3} + 1)$

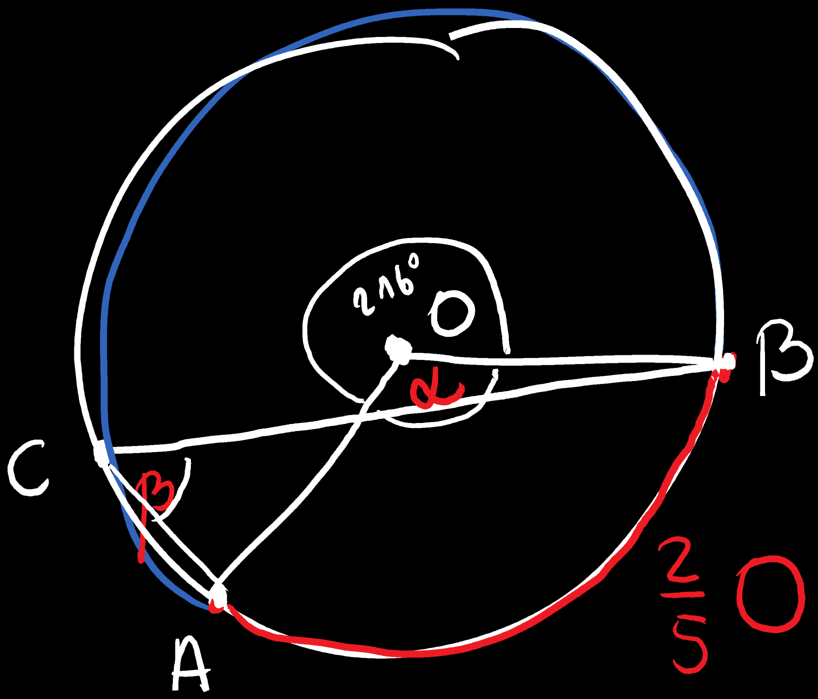
$$\text{tg}\, 30° = \frac{4}{x}$$

$$\frac{\sqrt{3}}{3} = \frac{4}{x}$$

$$12 = \sqrt{3}\, x \quad / : \sqrt{3}$$

$$\frac{12}{\sqrt{3}} = x$$

$$x = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$




$$\alpha + \beta = ?$$

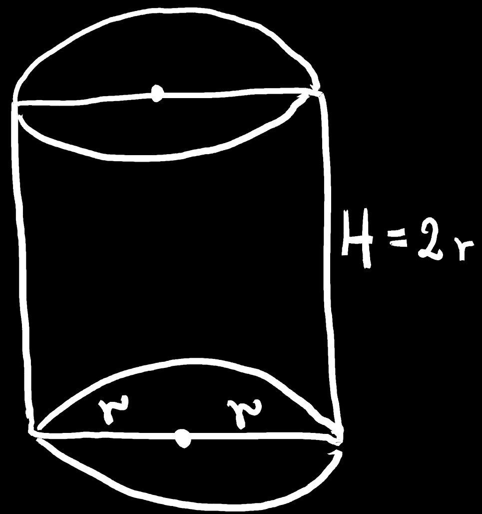
$$O - 360^\circ$$

$$\frac{2}{5}O - \alpha$$

$$\alpha = \frac{\frac{2}{\cancel{5}_{1}}\ \cancel{360}^{72}{}^\circ}{1} = 144^\circ$$

$$\beta = 72^\circ$$

$$\alpha + \beta = 144^\circ + 72^\circ = 216^\circ$$




$$H = 2r$$

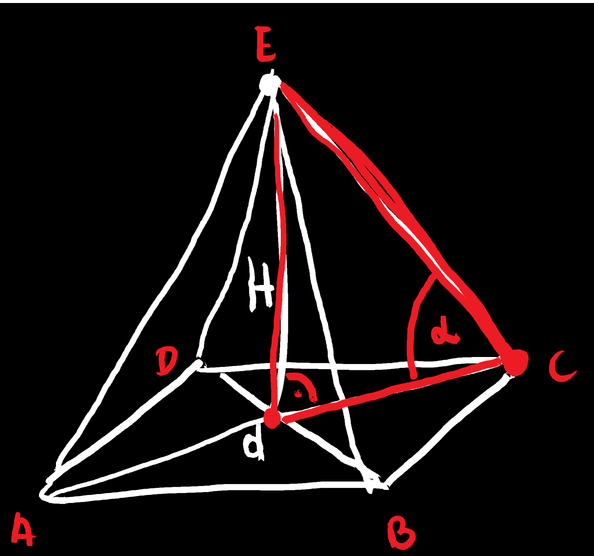
$$P_c = 2\pi r(r+H) = 2\pi r(r+2r) = 2\pi r \cdot 3r =$$
$$= 6\pi r^2$$

$$P_b = 2\pi r H = 2\pi r \cdot 2r = 4\pi r^2$$

$$\frac{P_c}{P_b} = \frac{6\cancel{\pi r^2}}{4\cancel{\pi r^2}} = \frac{3}{2} \quad / \cdot P_b$$

$$P_c = \frac{3}{2} \cdot P_b$$

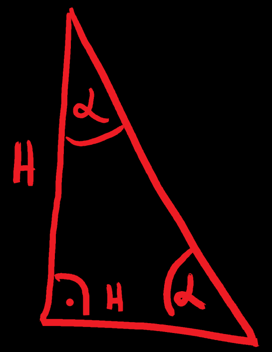
C




$d = 2H$

$\alpha = ?$

$\alpha = 45^\circ$




$\frac{1}{2} d = \frac{1}{2} \cdot 2H = H$

$$2, 3, 4, 5, 6, \underset{\underset{4}{\parallel}}{x}$$

$$\bar{x} = 4$$

(?) mediana $= 4$

$$2, 3, \underbrace{4, 4}, 5, 6$$

$$\bar{x} = \frac{2+3+4+5+6+x}{6}$$

$$4 = \frac{20+x}{6} \quad / \cdot 6$$

$$24 = 20 + x$$

$$24 - 20 = x$$

$$4 = x$$

$x$ - liczba chłopców

$3x$ - liczba dziewczyn

$4x$ - liczba uczniów

$$P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{x}{4x} = \frac{1}{4}$$

B

$\overline{\overline{A}} = x$

$\overline{\overline{\Omega}} = 4x$

$$f(x) = \underbrace{-4x^2 + x + 5}$$

$$g(x) = \underbrace{-4x + 6}$$

$$f(x) > g(x)$$

$$-4x^2 + x + 5 > -4x + 6$$

$$-4x^2 + x + 5 + 4x - 6 > 0$$

$$\underbrace{-4x^2 + 5x - 1 > 0}$$

$$\Delta = b^2 - 4ac$$

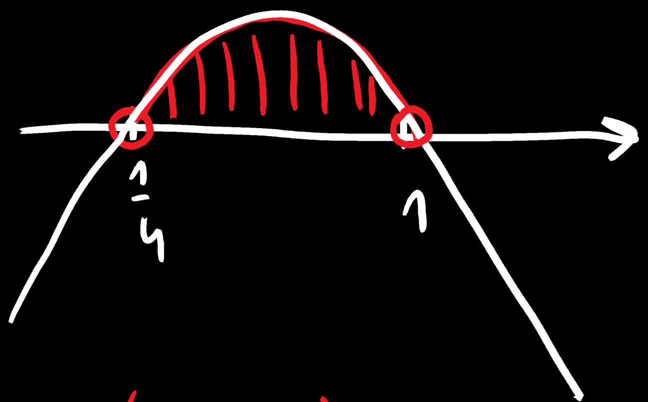
$$\Delta = 5^2 - 4 \cdot (-4) \cdot (-1)$$

$$\Delta = 25 - 16 = 9 > 0$$

$$\sqrt{\Delta} = 3$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-5 + 3}{-8} = \frac{-2}{-8} = \frac{1}{4}$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-5 - 3}{-8} = \frac{-8}{-8} = 1$$




$$\underbrace{x \in \left(\frac{1}{4}, 1\right)}$$

$A(-2,0)$ with $x_A = -2$, $y_A = 0$

$B(8,0)$ with $x_B = 8$, $y_B = 0$

$C(0,y)$ with $x_C = 0$, $y_C = y$

$\Delta$ - prostokątny

I

$$|AC| = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}$$

$$|AC| = \sqrt{(0+2)^2 + (y-0)^2}$$

$$|AC| = \sqrt{4 + y^2}$$

$$|BC| = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}$$

$$|BC| = \sqrt{(0-8)^2 + (y-0)^2} = \sqrt{64 + y^2}$$

II

$$|AC|^2 + |BC|^2 = |AB|^2$$

$$\sqrt{4+y^2}^{\,2} + \sqrt{64+y^2}^{\,2} = 10^2$$

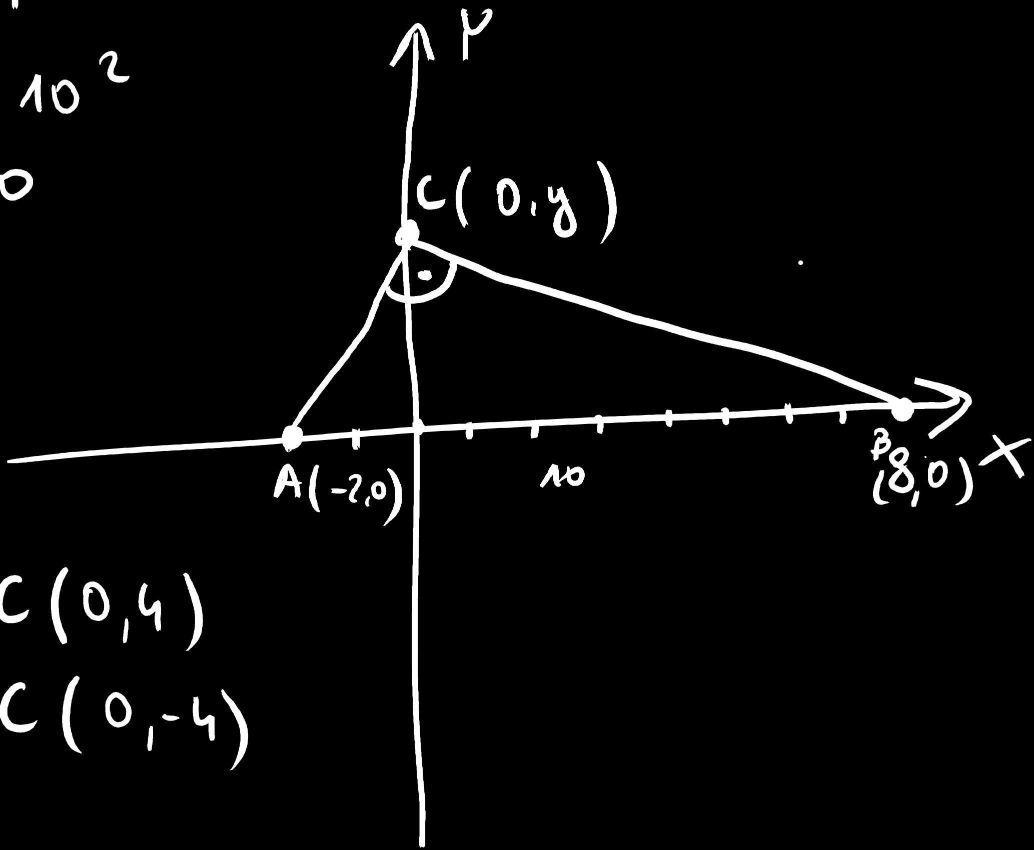
$$4 + y^2 + 64 + y^2 = 100$$

$$2y^2 + 68 = 100$$

$$2y^2 = 32 \quad / : 2$$

$$y^2 = 16 \quad / \sqrt{\phantom{x}}$$

$$y = 4 \qquad y = -4$$




III

$C(0,4)$

$C(0,-4)$

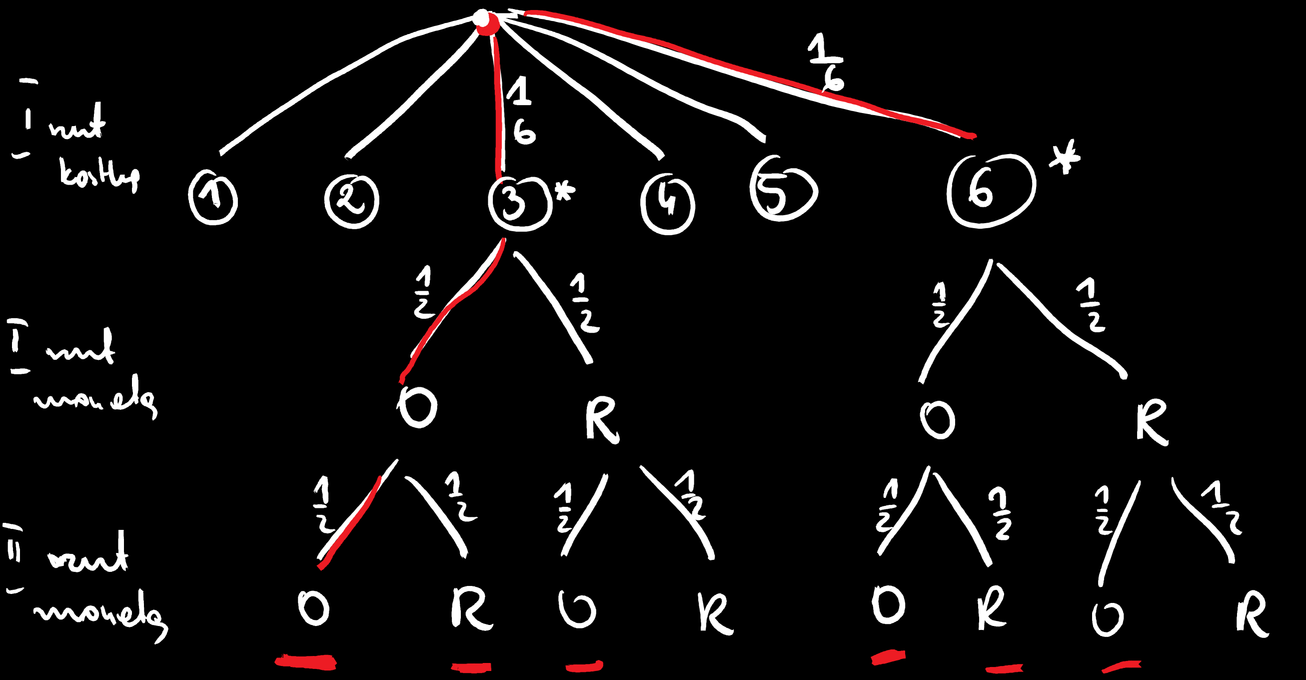
Doświadczenie losowe polega na jednoczesnym rzucie symetryczną sześcienną kostką do gry i dwiema symetrycznymi monetami. Oblicz prawdopodobieństwo otrzymania na kostce liczby oczek podzielnej przez 3, a na monetach – co najmniej jednego orła.

1 kostka

2 monety

1. orzeł / 3

1 O lub 2 O

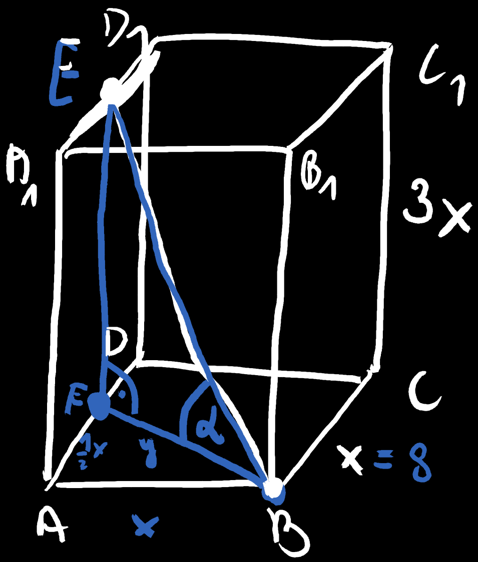



$$P(A) = \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 6 = \frac{1}{4}$$

II $A = \{(3,O,O), (3,O,R), (3,R,O), (6,O,O), (6,O,R)\ (6,R,O)\}$ $\quad \overline{\overline{A}} = 6$

$\overline{\overline{\Omega}} = \underset{K}{6} \cdot \underset{M}{2} \cdot \underset{M}{2} = 24$ $\quad P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{6}{24} = \frac{1}{4}$

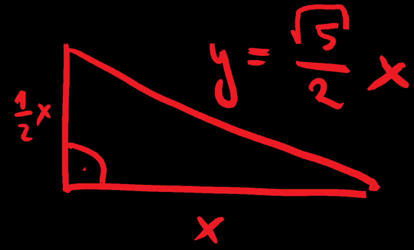
W graniastosłupie prawidłowym czworokątnym o podstawach ABCD i A B1 1C1 1 D (jak na rysunku) krawędź boczna jest trzy razy dłuższa od krawędzi podstawy. Z wierzchołka B poprowadzono odcinek BE, którego koniec E jest środkiem krawędzi A D1 1. Długość BE jest równa 4 41. Oblicz objętość graniastosłupa i wyznacz sinus kąta nachylenia odcinka BE do płaszczyzny podstawy graniastosłupa.




$|BE| = 4\sqrt{41}$

$V = ?$

I




$\left(\frac{1}{2}x\right)^2 + x^2 = y^2$

$\frac{1}{4}x^2 + 1x^2 = y^2$

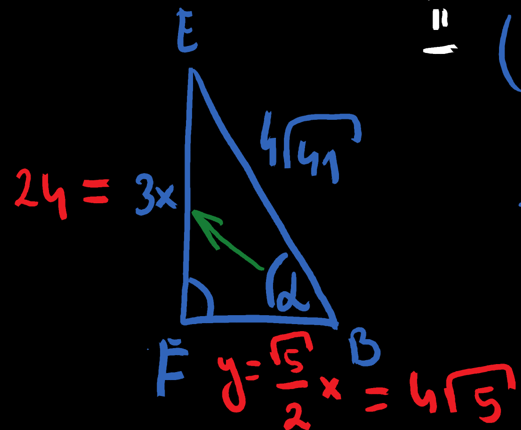
$\frac{5}{4}x^2 = y^2 \quad /\sqrt{\ }$

$\frac{\sqrt{5}}{2}x = y$

$y = \frac{\sqrt{5}}{2} \cdot 8$

$y = 4\sqrt{5}$

II




$(3x)^2 + \left(\frac{\sqrt{5}}{2}x\right)^2 = (4\sqrt{41})^2$

$\frac{9}{1}x^2 + \frac{5}{4}x^2 = 16 \cdot 41$

$\frac{36}{4}x^2 + \frac{5}{4}x^2 = 16 \cdot 41$

$\frac{41}{4}x^2 = 16 \cdot 41 \quad /:41$

III

$\sin\alpha = ?$

$\frac{1}{4}x^2 = 16 \quad / \cdot 4$

$x^2 = 64$

$x = 8$

V

$V = x^2 \cdot 3x$

$V = 8^2 \cdot 3 \cdot 8$

$V = 64 \cdot 24$

$V = 1536$

IV

$\sin\alpha = \frac{24}{4\sqrt{41}}$

$\sin\alpha = \frac{6}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}}$

$\sin\alpha = \frac{6\sqrt{41}}{41}$

29. $x^2 + y^2 + 11 > 2x + 6y$

$(x^2 - 2x + \underbrace{1^2}_{1}) + (y^2 - 6y + \underbrace{3^2}_{9}) + 1 > 0$

$a^2 - \underbrace{2ab}_{} + b^2 = (a-b)^2$

$a = x$

$b = 1$

$\underbrace{(x-1)^2}_{\geq 0} + \underbrace{(y-3)^2}_{\geq 0} + 1 > 0$

c.n.d

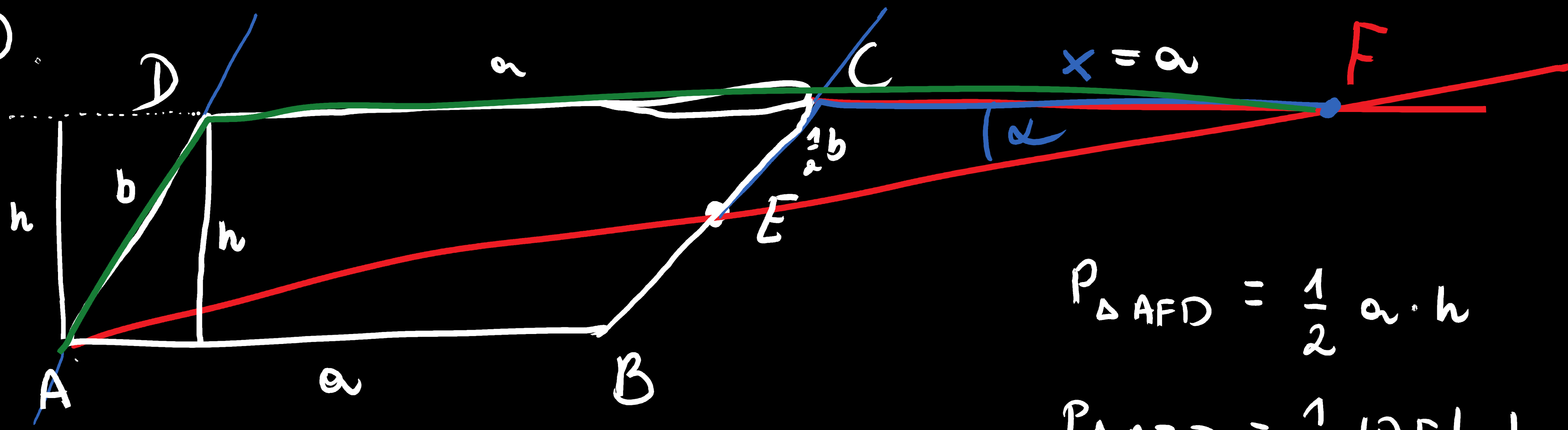
$(a-b)^2 \geq 0$

$(a+b)^2 \geq 0$

$(a-b)^2 + (a+b)^2 \geq 0$

$\underbrace{(a-b)^2}_{\geq 0} + 1 > 0$

30.




$$P_{\triangle AFD} = \frac{1}{2}a \cdot h$$

$$P_{\triangle AFD} = \frac{1}{2}|DF| \cdot h =$$

$$= \frac{1}{2} \cdot 2a \cdot h = \underline{a \cdot h}$$

$$\frac{x}{\frac{1}{2}b} = \frac{a+x}{b} \quad / \cdot b$$

$$\frac{x}{\frac{1}{2}} = a + x \quad / \cdot \frac{1}{2}$$

$$x = \frac{1}{2}(a+x)$$

$$P_{ABCD} = P_{\triangle AFD}$$

$$P_{ABCD} = \underbrace{a \cdot h} = |AB| \cdot h$$

$$x = \frac{1}{2}a + \frac{1}{2}x$$

$$\frac{1}{2}x = \frac{1}{2}a \qquad x = a$$

31. $\underline{x = -2}$

$a, b = ?$

$f(x) = ax^2 - 8x + c \qquad P(2, 2)$

$p = -\frac{b}{2a} \qquad p = -2 \qquad b = -8$

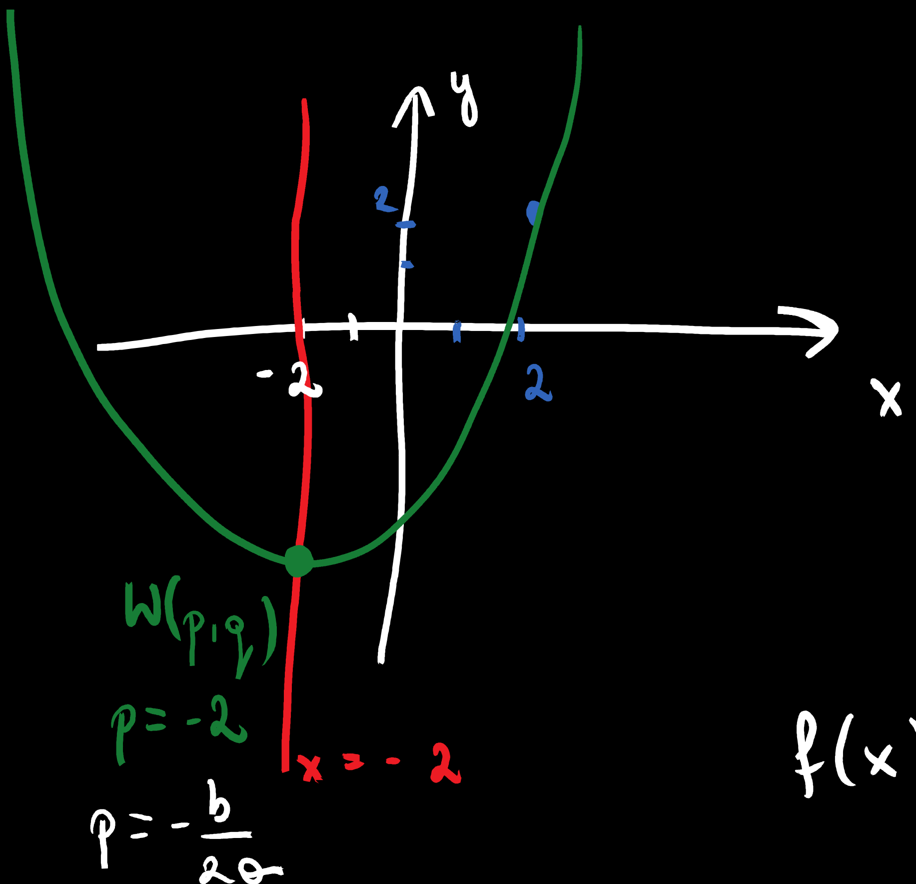
$-2 = -\frac{b}{2a}$

$-2 = \frac{8}{2a} \quad / \cdot 2a$

$-4a = 8$

$a = -2$

$f(x) = -2x^2 - 8x + c$



31 c. d.

$$f(x) = -2x^2 - 8x + c$$

$$P(2, 2)$$

$$x \quad f(x)$$

$$2 = -2 \cdot 2^2 - 8 \cdot 2 + c$$

$$2 = -8 - 16 + c$$

$$2 = -24 + c$$

$$26 = c$$

$$\begin{cases} a = -2 \\ c = 26 \end{cases}$$

32. $a_n = a_1 + (n-1)r$

$n = 8$

$S_8 = 236$

$a_1 = ?$  $a_8 = ?$

$a_1 - 3 = a_2$

$r = -3$

$$S_n = \frac{2a_1 + (n-1)\cdot r}{2}\cdot n$$

$$\underline{236} = \frac{2a_1 + 7\cdot(-3)}{\cancel{2}_1}\cdot \cancel{8}^4 \quad / :4$$

$$59 = 2a_1 - 21$$

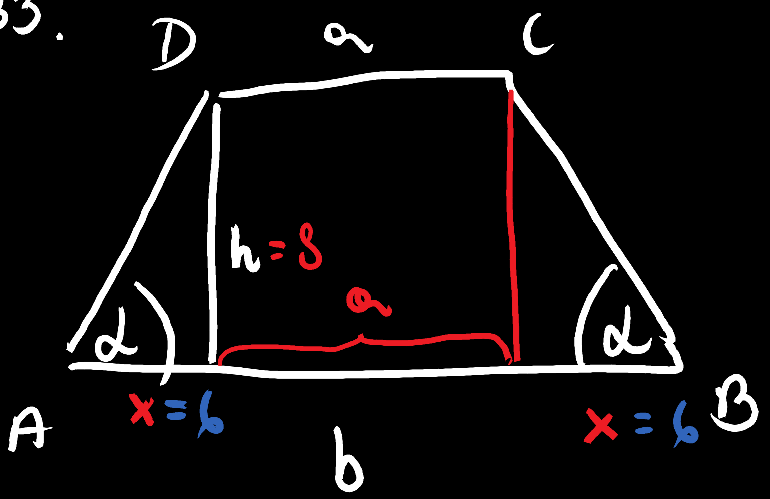
$$80 = 2a_1$$

$$40\,km = a_1$$

$$a_8 = 40 + 7\cdot(-3)$$

$$a_8 = 40 - 21 = 19\,km$$

33.




$$P = \frac{a+b}{2} \cdot h$$

$$80 = \frac{20}{2} \cdot h$$

$$80 = 10h$$

$$8 = h$$

$$a + b = 20$$

$$P = 80$$

$$\operatorname{tg}\alpha = \frac{4}{3}$$

$a = ?$

$b = ?$

$$\operatorname{tg}\alpha = \frac{h}{x}$$

$$\frac{4}{3} = \frac{8}{x}$$

$$4x = 24$$

$$x = 6$$

$$b = a + 2x$$

$$b = a + 2 \cdot 6$$

$$b = a + 12$$

$$a + a + 12 = 20$$

$$2a = 8$$

$$\boxed{a = 4}$$

$$\boxed{b = 4 + 12 = 16}$$