1. $\log_2 \frac{1}{\sqrt{8}} = \log_2 \frac{1}{(2^3)^{\frac{1}{2}}} = \log_2 \frac{1}{2^{\frac{3}{2}}} =$

$$= \left(\frac{1}{2}\right)^{\frac{3}{2}}$$

$$= \log_{\textcircled{2}} \underbrace{2^{-\frac{3}{2}}} = -\frac{3}{2}$$

$$2^{?} = 2^{-\frac{3}{2}}$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\sqrt[3]{a} = a^{\frac{1}{3}}$$

$$\frac{1}{a} = a^{-1}$$

$$\log_a a^n = n$$

Ⓐ

2.

$$a = \frac{14\sqrt{2}}{\underset{a}{\sqrt{2}} \underset{-}{-} \underset{b}{3}} = \frac{14\sqrt{2}\cdot(\sqrt{2}+3)}{(\sqrt{2}-3)\cdot(\sqrt{2}+3)} = \frac{28+42\sqrt{2}}{\sqrt{2}^2 - 3^2} =$$

$$(a-b)(a+b) = \underbrace{a^2-b^2}$$

↑ ↑ ↑ ↑

$1{,}41 \cdot 6 \approx 8{,}46$

$$= \frac{28+42\sqrt{2}}{2-9} = \frac{-7(-4-6\sqrt{2})}{-7} = -4-6\sqrt{2} \approx -12{,}46$$

Ⓑ

$a^2 = a \cdot a$

$a^3 = a \cdot a \cdot a$

3.

$$\underline{x} = \underbrace{9k + 7} \qquad k \in \mathbb{C} \qquad 70 = 9 \cdot \underbrace{7} + 7$$

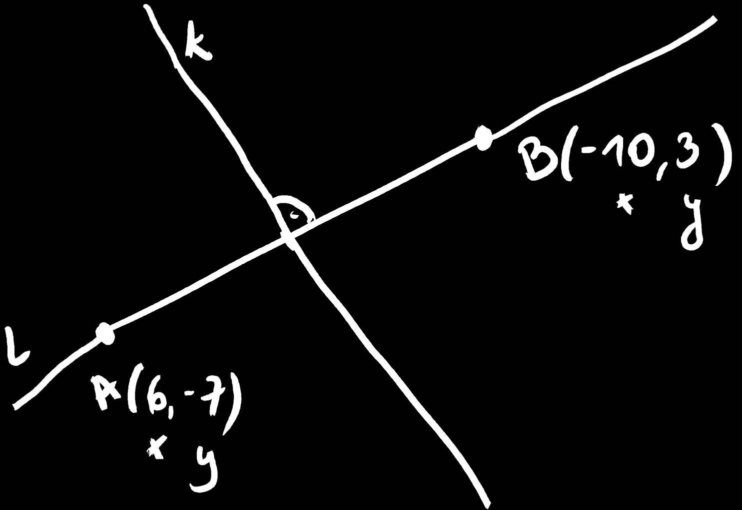
$$\underbrace{x^2} = (\underbrace{9k}_{a} + \underbrace{7}_{b})^2 = \qquad 25 = \underbrace{9} \cdot 2 + \underbrace{7}$$

$$= (9k)^2 + 2 \cdot 9k \cdot 7 + 7^2 = \qquad (a+b)^2 = a^2 + 2ab + b^2$$

$$= 81k^2 + \underbrace{9 \cdot 14k} + 49 = \underline{9} \cdot \underbrace{(9k^2 + 14k + 5)}_{\in \mathbb{C}} + \underbrace{4}$$

Ⓑ

4. Prosta l przechodzi przez punkty A= (6, -7) , ,B = (− 10,3) . Prosta k jest symetralną odcinka AB. Współczynnik kierunkowy prostej k jest równy



$k \perp l$: $\quad a_1 \cdot a_2 = -1$

$-\frac{5}{8} \cdot a_2 = -1 \quad / \cdot \left(-\frac{8}{5}\right)$

$$\boxed{a_2 = \frac{8}{5}}$$

$l: \quad y = ax + b_1$

$$\begin{cases} -7 = a \cdot 6 + b \\ 3 = a \cdot (-10) + b \end{cases}$$

$$\begin{cases} -7 = 6a + b \\ 3 = -10a + b \quad / \cdot (-1) \end{cases}$$

$$\begin{cases} -7 = 6a + b \\ -3 = 10a - b \end{cases}$$

$-10 = \underbrace{16a}_{} \quad / : 16$

$-\frac{10}{16} = a$

$$\boxed{a = -\frac{5}{8}}$$

$k: \quad y = \underbrace{\frac{8}{5}}_{} x + b_2$

Ⓑ

5. $a_n = \frac{2n+1}{n+3}$

$a_3 = \frac{2\cdot 3+1}{3+3} = \frac{7}{6}$

$a_5 = \frac{2\cdot 5+1}{5+3} = \frac{11}{8}$

Ⓐ

a_3, a_5

$(a_3, x, a_5) \qquad x = ?$

$(\frac{7}{6}, \underset{\downarrow}{x}, \frac{11}{8})$ - ciąg arytm.

$a_{n-1} \quad a_n \quad a_{n+1}$

$a_n = \frac{a_{n-1}+a_{n+1}}{2}$

$x = \frac{\frac{7}{6}+\frac{11}{8}}{2} = \frac{\frac{28}{24}+\frac{33}{24}}{\frac{2}{1}} = \frac{61}{24}\cdot\frac{1}{2}$

$x = \frac{61}{48}$

6. $y = x^2 - 4\sqrt{3}x + 12$

$\Delta = b^2 - 4ac$

$\Delta = (-4\sqrt{3})^2 - 4 \cdot 1 \cdot 12$

$\Delta = 16 \cdot 3 - 48$

$\Delta = 0$

$x_0 = \frac{-b}{2a} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

$x_0^3 = ?$

$(2\sqrt{3})^3 = 8 \cdot 3\sqrt{3} = 24\sqrt{3}$

$\underbrace{\sqrt{3} \cdot \sqrt{3}}_{} \cdot \sqrt{3}$

Ⓒ

7. $f(x) = \left(\frac{1}{4}\right)^{x}$

$x = -\frac{1}{2}$

$x = 2$

$f\left(-\frac{1}{2}\right) = \left(\frac{1}{4}\right)^{-\frac{1}{2}} = 4^{\frac{1}{2}} =$

$= \sqrt{4} = 2$

Ⓑ

8. Dany jest ciąg geometryczny o wyrazach różnych od 0. Suma siódmego i ósmego wyrazu tego ciągu jest równa 0. Oznacza to, że suma tysiąca początkowych wyrazów tego ciągu jest równa:

$a_7 + a_8 = 0$

$\Downarrow$

$a_7 \cdot q = a_8$

$q = \frac{a_8}{a_7}$

$q = -1$

$S_{1000} = 0$

$a_7 = 8$ $\quad a_8 = -8$ $\quad a_9 = 8$

$a_1 = 8$

$a_2 = -8$

$a_3 = 8$

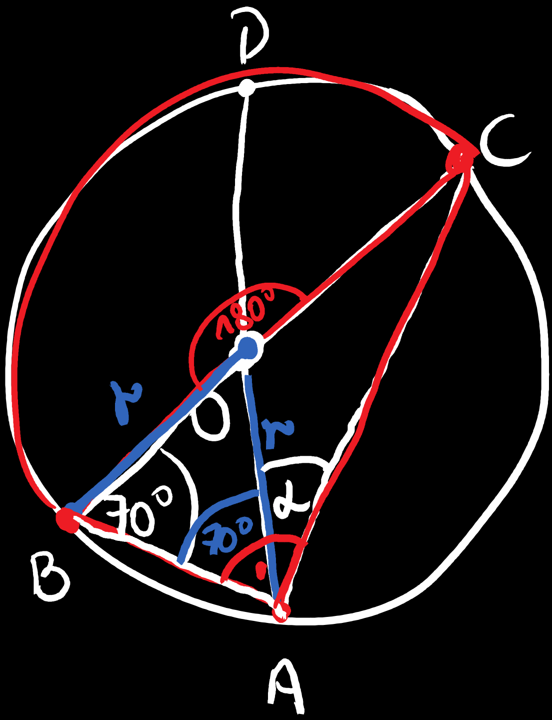
$a_1 = -8$

$a_2 = 8$

$\vdots$

Ⓓ

9.



$\alpha = ?$

$\alpha = 90° - 70° = 20°$

D

10. $\triangle ABC \sim \triangle DEF$

$O_{\triangle ABC} = 16$

$P_{\triangle ABC} = 12$

$P_{\triangle DEF} = 60$

$O_{\triangle DEF} = ?$

$$k^2 = \frac{P_{ABC}}{P_{DEF}} = \frac{12}{60} = \frac{2}{10} = \frac{1}{5}$$

$$k^2 = \frac{1}{5} \quad / \sqrt{\ } \qquad k = \frac{1}{\sqrt{5}}$$

$$\rightarrow \frac{O_{ABC}}{O_{DEF}} = k \qquad \frac{16}{O_{DEF}} = \frac{1}{\sqrt{5}}$$

$$O_{DEF} = 16\sqrt{5}$$

Ⓑ

11. $f(x) = (4m-2)\underset{\uparrow}{x} + k - 3$

II , IV

$y = ax + \underbrace{b}$

$a = 4m - 2$

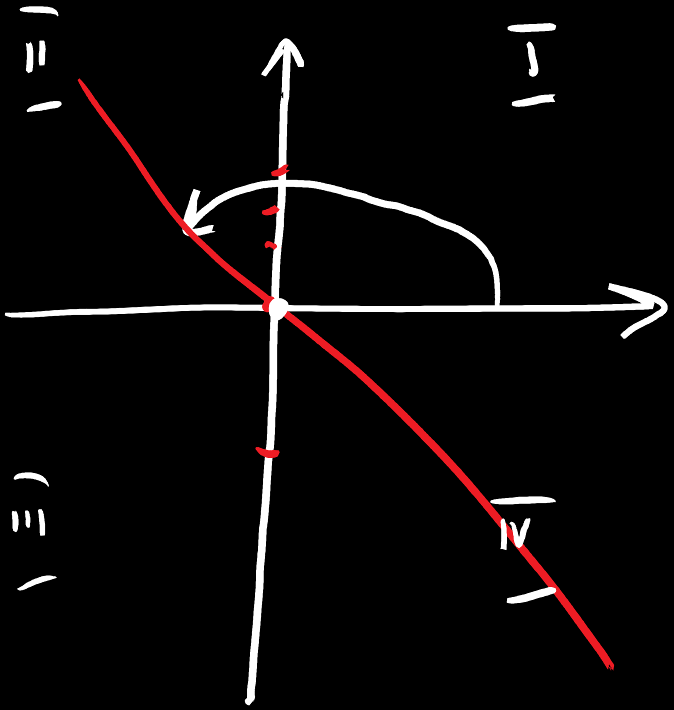
$b = k - 3$

$b = 0$

$y = ax$

$0 = k - 3$

$\underline{3 = k}$



$a < 0$

$\underbrace{4m-2} < 0$

$4m < 2$

$\underline{m < \frac{1}{2}}$

$$\begin{cases} m < \frac{1}{2} \\ k = 3 \end{cases}$$

$\textcircled{C}$

12. sym OX

$f(x) = x^2 - 4$

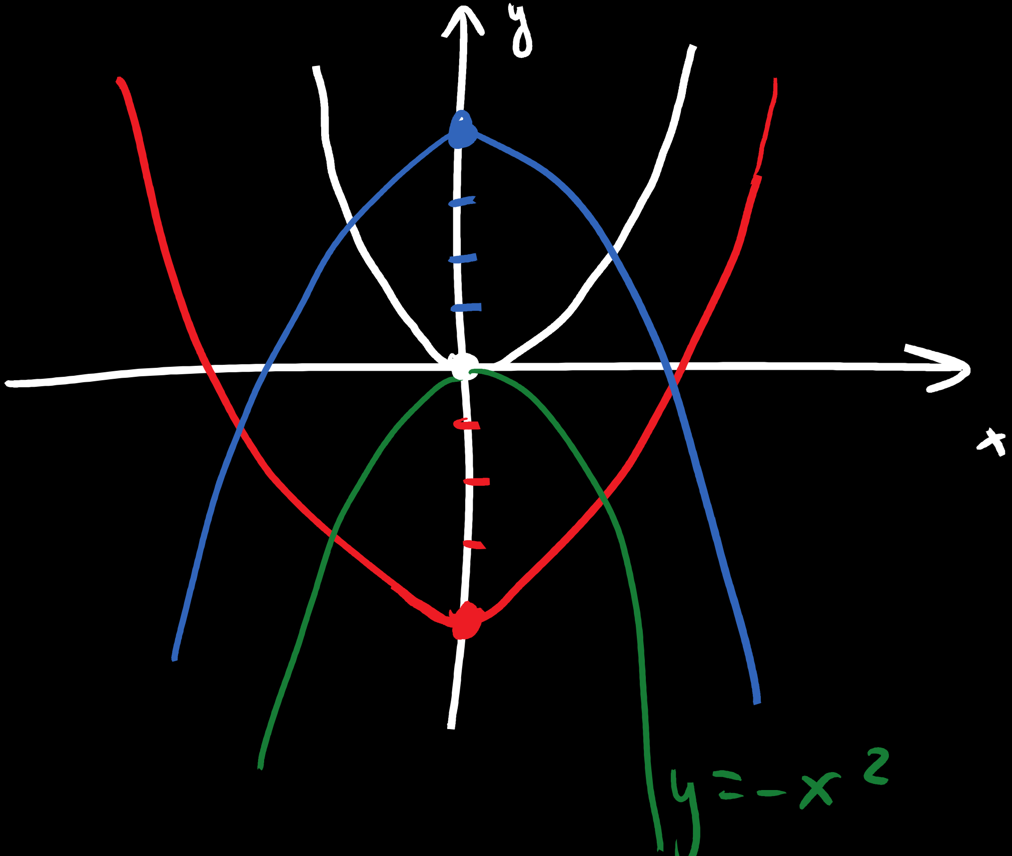
$\boxed{y=ax^2}$

$\boxed{y=\underline{1}x^2}$

$y = -1x^2 + 4$

$y = -x^2 + 4$

Ⓒ



13. $W = \dfrac{x-3}{x^2-4x+4}$

$x^2-4x+4 \neq 0$

$(a-b)^2 = a^2 - 2ab + b^2$

$(x-2)^2 \neq 0$

$x-2 \neq 0$

$\underbrace{x \neq 2}$

$D_f = \mathbb{R} \setminus \{2\}$

Ⓒ

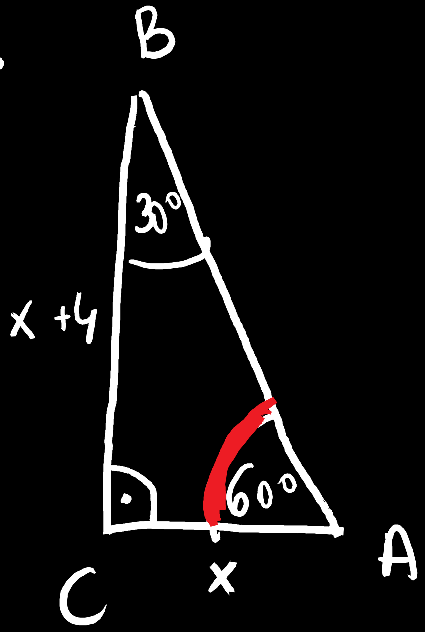
1° $\dfrac{1}{a}$, $a \neq 0$

2° $\sqrt{a}$, $a \geqslant 0$

$\sqrt{a} = b \qquad \underset{\substack{\geqslant \\ 0}}{\underbrace{b^2}} = \underset{\substack{\geqslant \\ 0}}{a}$

3° $\dfrac{1}{\sqrt{a}}$, $a > 0$

14.



$x = ?$

$$tg\, 60° = \frac{x+4}{x}$$

$$\sqrt{3} = \frac{x+4}{\boxed{x}} \quad / \cdot x$$

$$\sqrt{3}x = x+4$$

$$\sqrt{3}x - x = 4$$

$$x \cdot (\sqrt{3} - 1) = 4 \quad / : (\sqrt{3} - 1)$$

$$x = \frac{4 \cdot (\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{4(\sqrt{3} + 1)}{\sqrt{3}^2 - 1}$$

$$x = \frac{4(\sqrt{3} + 1)}{3 - 1}$$

$$x = \frac{\overset{2}{\cancel{4}}(\sqrt{3} + 1)}{\underset{1}{\cancel{2}}}$$

$$x = 2(\sqrt{3} + 1)$$

$$(a-b)(a+b) = a^2 - b^2$$

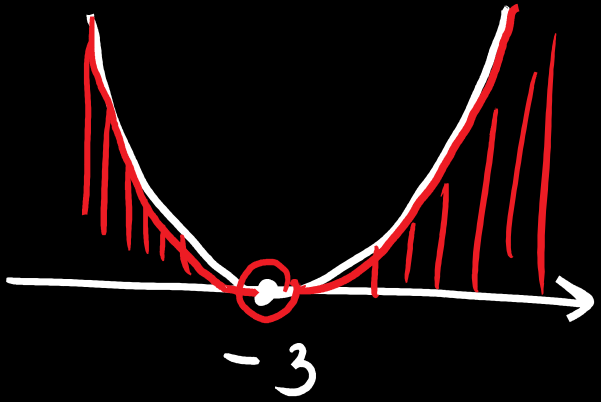
Ⓓ

15. $(3x+9)^2 > 0$

$3x+9=0$

$3x=-9$

$\underbrace{x=-3}$

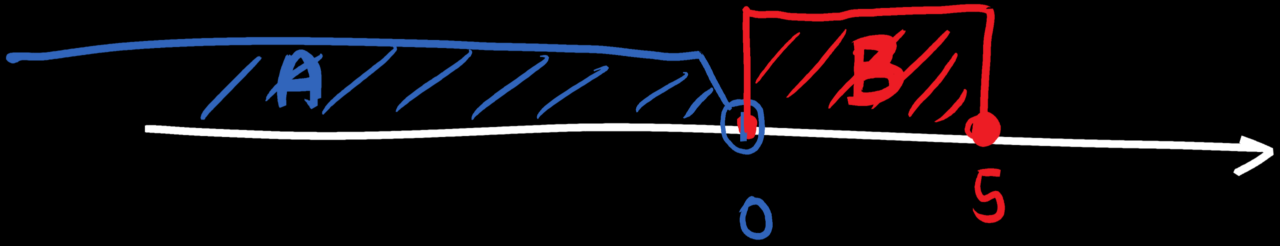


$x \in \mathbb{R} \setminus \{-3\}$

16. $A = (-\infty, 0)$ $B = \langle 0, 5 \rangle$

$B \setminus A$

$B - A$



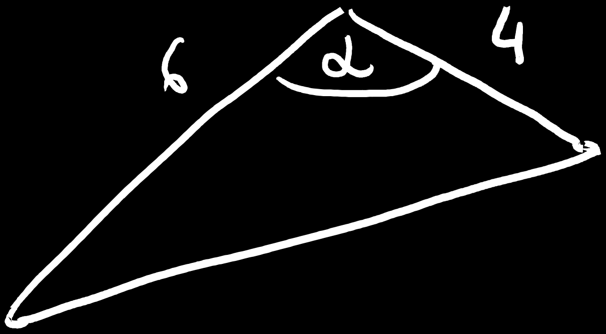
$\cup$ - suma

$\cap$ - część wspólna (iloczyn)

$\underline{B} \setminus \overset{\downarrow}{A} = \langle 0, 5 \rangle = B$

Ⓟ

17.



$P = \frac{1}{2} a \cdot b \sin \gamma$

$3\sqrt{15} = \frac{1}{2} \cdot 6 \cdot 4 \cdot \sin \alpha$

$3\sqrt{15} = 12 \cdot \underline{\sin \alpha} \quad /:12$

$\frac{3\sqrt{15}}{12} = \sin \alpha$

$\alpha > 90°$

$P = 3\sqrt{15}$

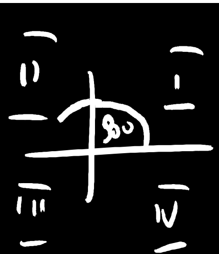
$\boxed{\cos \alpha = ?}$

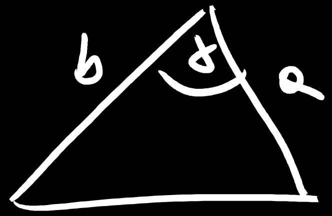
$\sin \alpha = \frac{\sqrt{15}}{4}$

$\sin^2 \alpha + \cos^2 \alpha = 1$

$\left(\frac{\sqrt{15}}{4}\right)^2 + \cos^2 \alpha = 1$

$\frac{15}{16} + \cos^2 \alpha = 1$





$\cos^2 \alpha = 1 - \frac{15}{16}$

$\cos^2 \alpha = \frac{1}{16} \quad /\sqrt{\ }$

$\cos \alpha = \frac{1}{4}$

lub

$\boxed{\cos \alpha = -\frac{1}{4}}$

18. $\underbrace{\text{4 - rzuty}}$ A - orzeł $\underline{1}$ lub O

$P(A) = \frac{\bar{\bar{A}}}{\bar{\bar{\Omega}}}$

ORRO ROOR

$\bar{\bar{\Omega}} = \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 16$

$A = \{(O,R,R,R), (R,O,R,R)(R,R,O,R)(R,R,R,O)(R,R,R,R)\}$

$\bar{A} = 5$

$P(A) = \frac{5}{16}$

Ⓑ

20 . $S_{\underset{\uparrow}{n}} = 3n^2+4n$ $\qquad a_5 = ?$

$$S_5 = \underbrace{a_1+a_2+a_3+a_4}+\boxed{a_5}$$

$$S_4 = a_1+a_2+a_3+a_4$$

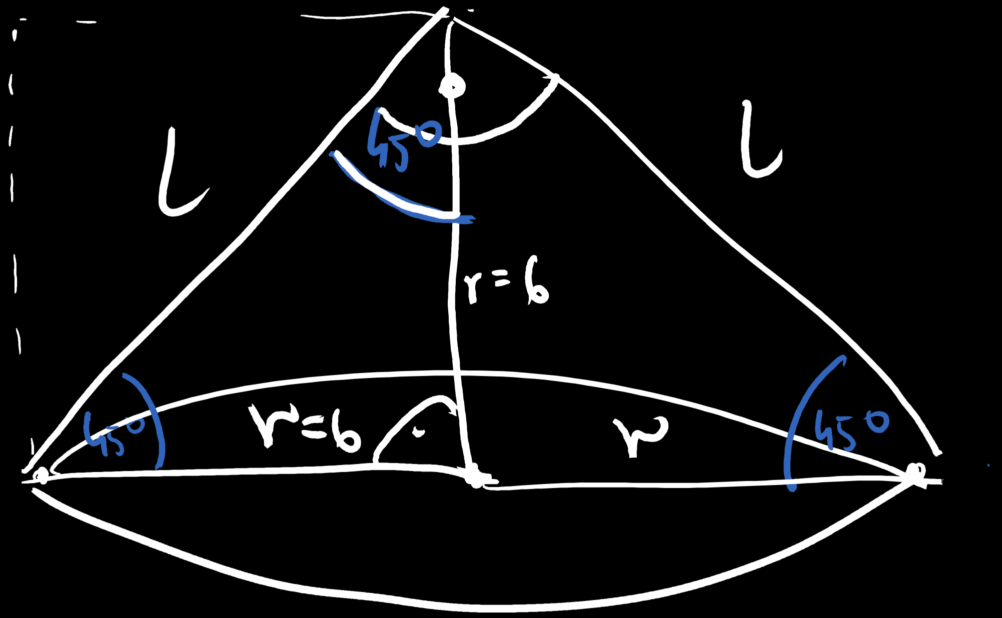
$$S_{\underset{\uparrow}{5}} = 3\cdot 5^2+4\cdot 5 = 75+20 = 95$$

$$S_4 = 3\cdot 4^2+4\cdot 4 = 48+16 = 64$$

$$a_5 = S_5 - S_4 = 95-64 = 31$$

Ⓑ

19.



$r = 6$

$P_c = ?$

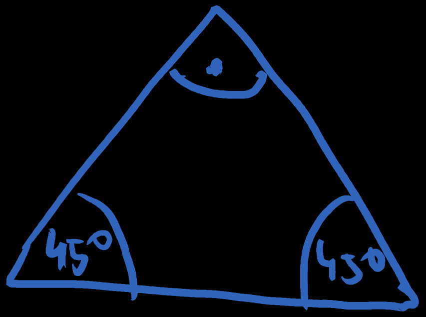
$P_c = \pi r(r + l)$

$l = a\sqrt{2}$

$l = r\sqrt{2}$

$l = 6\sqrt{2}$

Ⓑ

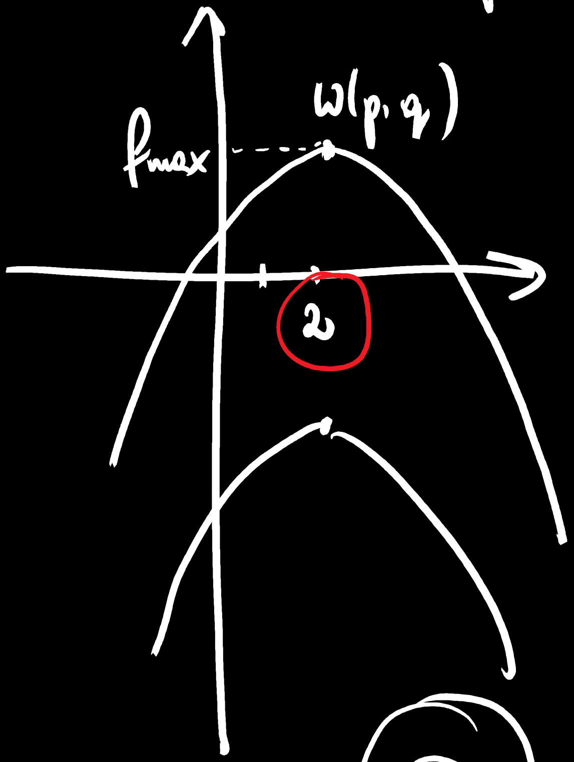


$P_c = \pi \cdot 6 \cdot (6 + 6\sqrt{2}) = \pi \cdot 6 \cdot 6(1+\sqrt{2}) = 36\pi(1+\sqrt{2})$

21. $\underset{\substack{\uparrow\\ \text{zmienna}}}{f(x)} = (\underset{\substack{\uparrow\\ \text{parametr.}}}{m}+3)x^2 + 16x + 5$

$x = 2$

$f_{max} = ?$



Ⓓ

$p = -\frac{b}{2a}$ $\quad q = \frac{-\Delta}{4a}$

$p = 2$

$2 = -\frac{16}{2\cdot(m+3)}$

$2 = \frac{-8}{m+3} \quad / \cdot (m+3)$

$2m+6 = -8$

$2m = -14$

$m = -7$

$f(x) = -4x^2+16x+5$

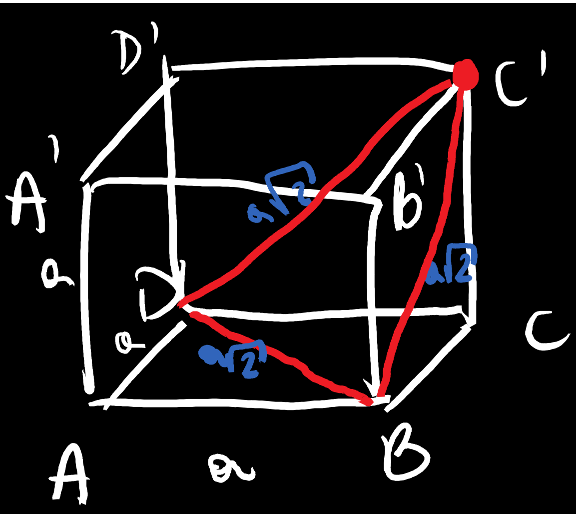
$f_{max} = f(p) = f(2) =$

$= -4\cdot 2^2 + 16\cdot 2 + 5 =$

$= -16+32+5 =$

$= 21$

22.



$$P_{\Delta BDC'} = \frac{(a\sqrt{2})^2\sqrt{3}}{4}$$

$$P_{\Delta BDC'} = \frac{a^2\cdot 2\sqrt{3}}{4} = \frac{a^2\sqrt{3}}{2}$$

Ⓑ

23. $x + \frac{1}{x} = 6 \quad / (\ )^2$

$x^2 + \frac{1}{x^2} = ?$

$\left(\underset{a}{x} + \underset{b}{\frac{1}{x}}\right)^2 = 36$

$(a+b)^2 = a^2 + 2ab + b^2$

$x^2 + 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 36$

$x^2 + 2 + \frac{1}{x^2} = 36 \quad / -2$

$x^2 + \frac{1}{x^2} = 34$

Ⓓ

24. $(\underset{a}{4x} - \underset{b}{1})^2 < (\underset{a}{2} - \underset{b}{5x})^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(4x)^2 - 2\cdot 4x \cdot 1 + 1^2 < 4 - 2\cdot 2\cdot 5x + (5x)^2$

$16x^2 - 8x + 1 < 4 - 20x + 25x^2$

$16x^2 - 8x + 1 - 4 + 20x - 25x^2 < 0$

$-9x^2 + 12x - 3 < 0 \quad /:3$

$\underbrace{\underbrace{-3}_{}x^2 + \underbrace{4}_{}x \underbrace{-1}_{} < 0}_{}$

$\Delta = b^2 - 4ac$

$\Delta = 16 - 4\cdot(-3)(-1)$

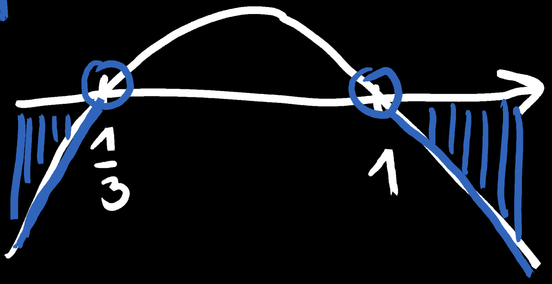
$\Delta = 16 - 12 = 4 \quad \sqrt{\Delta} = 2$

$x_1 = \frac{-b-\sqrt{\Delta}}{2a}$

$x_1 = \frac{-4-2}{-6} = 1$

$x_2 = \frac{-b+\sqrt{\Delta}}{2a}$

$x_2 = \frac{-4+2}{-6} = \frac{1}{3}$



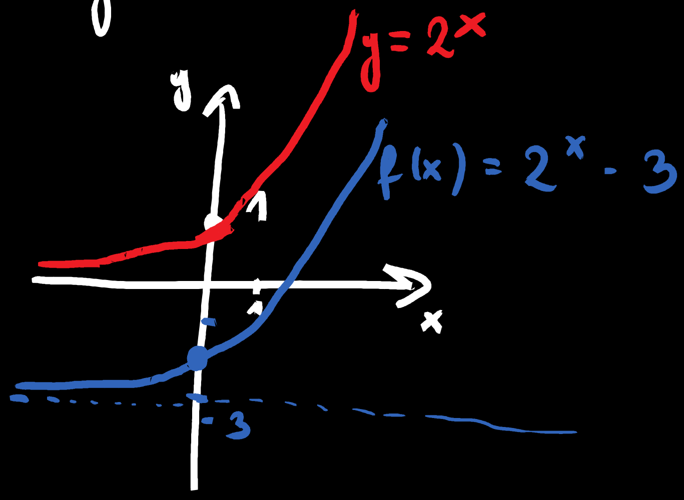
$x \in (-\infty, \frac{1}{3}) \cup (1, \infty)$

25. $f(x) = 2^x - 3$

$Z_w = ?$

$y = a^x$ $a > 1$ $a \in (0, 1)$

$Z_w = (-3, \infty)$



argumenty - x

wartości funkcji - y

26. z: $a < 1$

t: $\underbrace{\frac{1}{1-a}} \geq 4a \; / \cdot (1-a) > 0$

$1-a \neq 0$

$\underbrace{1-a > 0}$

$a^2 \geq 0$

$\underbrace{(a-b)^2 \geq 0}$

$(a+b)^2 \geq 0$

$(a+b)^2 + (a-b)^2 \geq 0$

$1 \geq 4a(1-a)$

$1 \geq 4a - 4a^2$

$4a^2 - 4a + 1 \geq 0$

$a^2 \quad 2ab \quad b^2$

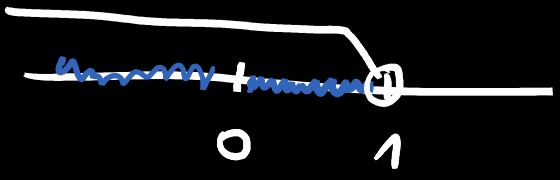
$a = 2a \quad \Downarrow \quad b = 1$

$2 \cdot 2a \cdot 1 = 4a$

$(2a-1)^2 \geq 0$

c.n.d.

$a < 1$



27. $f(x) = \underbrace{\,}x^2 + \underbrace{b}x + \underbrace{c}$

$x_1 = -4$

$x_2 = 2$

$b, c = ?$

$f(x) = a(x - x_1)(x - x_2)$

$f(x) = 1(x+4)(x-2)$

$f(x) = x^2 - 2x + 4x - 8$

$\underbrace{f(x) = x^2 + 2x - 8}$

$b = 2$

$c = -8$

28. $(\underset{a_{n-1}}{3^a}, \underset{a_n}{3^b}, \underset{a_{n+1}}{3^c})$ - ciąg geom.

a, b, c - ciąg arytm.

$(a^n)^m = a^{nm}$

$a^n \cdot a^m = a^{n+m}$

a_1, a_2, a_3

$a_1 \cdot q = a_2$

$a_2 \cdot q = a_3$

$q = \frac{a_2}{a_1}$

$q = \frac{a_3}{a_2}$

$\frac{a_2}{a_1} = \frac{a_3}{a_2}$

$a_2^2 = a_1 \cdot a_3$

$a_n^2 = a_{n-1} \cdot a_{n+1}$

$(3^b)^2 = 3^a \cdot 3^c$

$3^{2b} = 3^{a+c}$

$2b = a+c \quad /:2$

$b = \frac{a+c}{2}$

c.n.d

$a_n = \frac{a_{n-1} + a_{n+1}}{2}$

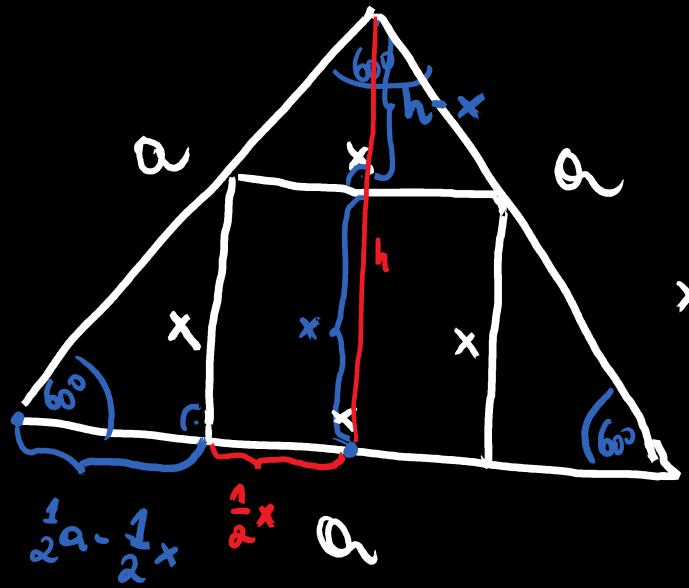
29. $P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{10}{216} = \frac{5}{108} \in \langle 0, 1 \rangle$

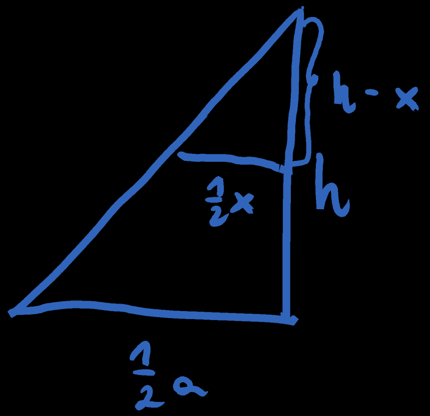
A - suma $\geqslant 16$

$\overline{\overline{\Omega}} = \underline{6} \cdot \underline{6} \cdot \underline{6} = 216$

$A = \{ (4,6,6)(6,4,6)(6,6,4)(5,6,6)(6,5,6)(6,6,5)(6,6,6),$
$(5,5,6)(5,6,5)(6,5,5) \}$

$\overline{\overline{A}} = 10$





$x = ?$

$h = \frac{a\sqrt{3}}{2}$

$$x = \frac{a^2\sqrt{3}}{a\sqrt{3}+2a}$$

$$x = \frac{\cancel{a}^2\sqrt{3}}{\cancel{a}(\sqrt{3}+2)}$$

$$x = \frac{a\sqrt{3}}{\sqrt{3}+2}$$

$$\frac{\frac{1}{2}x}{h-x} = \frac{\frac{1}{2}a}{h}$$

$$\frac{\cancel{\frac{1}{2}}x}{\frac{a\sqrt{3}}{2}-x} = \frac{\cancel{\frac{1}{2}}a}{\frac{a\sqrt{3}}{2}}$$

$$x \cdot \frac{a\sqrt{3}}{2} = a \cdot \left(\frac{a\sqrt{3}}{2} - x\right)$$

$$\frac{xa\sqrt{3}}{2} = \frac{a^2\sqrt{3}}{2} - ax \quad / \cdot 2$$

$$xa\sqrt{3} = a^2\sqrt{3} - 2ax$$

$$\underline{x}a\sqrt{3} + 2a\underline{x} = a^2\sqrt{3}$$

$$x \cdot (a\sqrt{3} + 2a) = a^2\sqrt{3} \quad / : (a\sqrt{3}+2a)$$

30. c. d.

$$x = \frac{a\sqrt{3} \cdot (\sqrt{3} - 2)}{(\sqrt{3} + 2) \cdot (\sqrt{3} - 2)} = \frac{3a - 2\sqrt{3}a}{3 - 4} = \frac{\cancel{-}a(-3 + 2\sqrt{3})}{-\cancel{1}} =$$

$$= (2\sqrt{3} - 3)a$$

31. $A(\underset{x_A}{4},\underset{y_A}{2})$ $B(\underset{x_B}{1},\underset{y_B}{-3})$ $|\sphericalangle ACB| = 90^\circ$

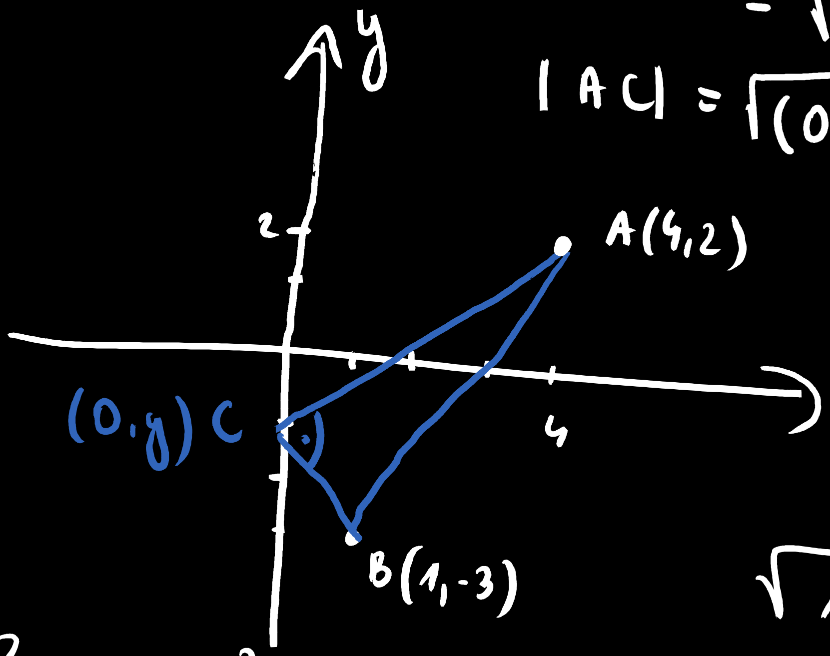
$C(\underset{x_C}{0},\underset{y_C}{y})$

$$|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(1-4)^2 + (-3-2)^2} =$$

$$= \sqrt{9+25} = \sqrt{34}$$

$$|AC| = \sqrt{(0-4)^2 + (y-2)^2} = \sqrt{16 + (y-2)^2}$$

$$|BC| = \sqrt{(0-1)^2 + (y+3)^2} = \sqrt{1 + (y+3)^2}$$



z tw. Pit.

$+$

$$|BC|^2 + |AC|^2 = |AB|^2$$

$$\sqrt{1+(y+3)^2}^{\,2} + \sqrt{16+(y-2)^2}^{\,2} = \sqrt{34}^{\,2}$$

$$1 + (y+3)^2 + 16 + (y-2)^2 = 34$$

$$(y+3)^2 + (y-2)^2 = 17$$

31.c.d

$$(y+3)^2 + (y-2)^2 = 17$$

$$y^2 + 6y + 9 + y^2 - 4y + 4 = 17$$

$$2y^2 + 2y + 13 - 17 = 0$$

$$2y^2 + 2y - 4 = 0 \quad / : 2$$

$$\boxed{y^2 + y - 2 = 0}$$

$$\Delta = b^2 - 4ac = 1 + 4 \cdot 2 = 9 > 0$$

$$\sqrt{\Delta} = 3$$

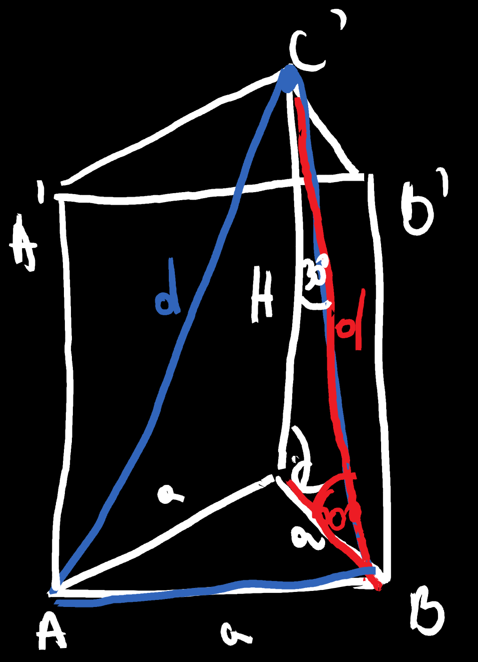
$$y_1 = \frac{-b - \sqrt{\Delta}}{2a} \qquad y_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

$$y_1 = \frac{-1 - 3}{2} = -2 \qquad y_2 = \frac{-1 + 3}{2} = 1$$

$$C(0, -2)$$

$$C(0, 1)$$

32.



$P_{\square} = 2\sqrt{3}$

$P_{\square} = a \cdot H$

$2\sqrt{3} = a \cdot H$

$H = a\sqrt{3}$

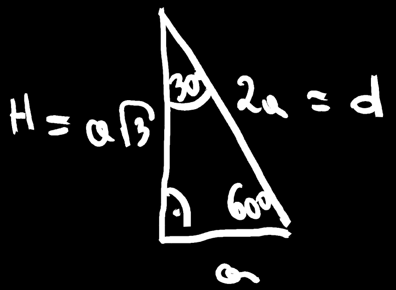
$2\sqrt{3} = a \cdot a\sqrt{3} \;/ :\sqrt{3}$

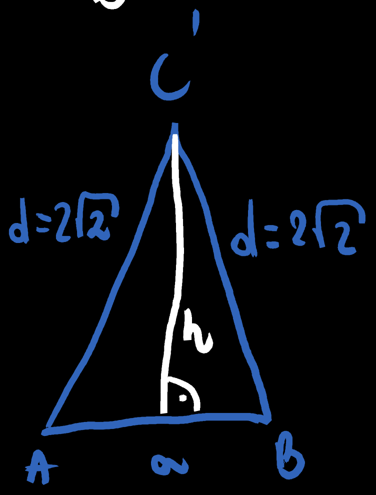
$2 = a^2 \;/ \sqrt{\;}$

$\boxed{\sqrt{2} = a}$

$d = 2\sqrt{2}$

$H = \sqrt{6}$





$P_{\triangle ABC'} = ?$

$P = \frac{a \cdot h}{2} = \frac{\sqrt{2} \cdot \frac{\sqrt{30}}{2}}{2} = \frac{\sqrt{60}}{4} = \frac{2\sqrt{15}}{4} = \frac{\sqrt{15}}{2}$

z tw. Pit.

$\left(\frac{1}{2}a\right)^2 + h^2 = d^2$

$\left(\frac{\sqrt{2}}{2}\right)^2 + h^2 = (2\sqrt{2})^2$

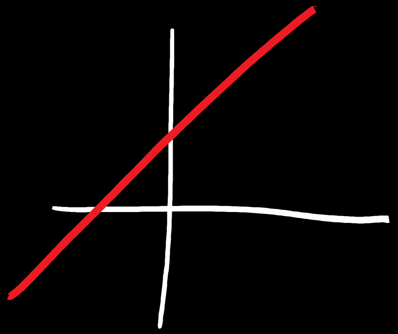
$\frac{2}{4} + h^2 = 4 \cdot 2$

$h^2 = 7\frac{1}{2} = \frac{15}{2} \;/ \sqrt{\;}$

$h = \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

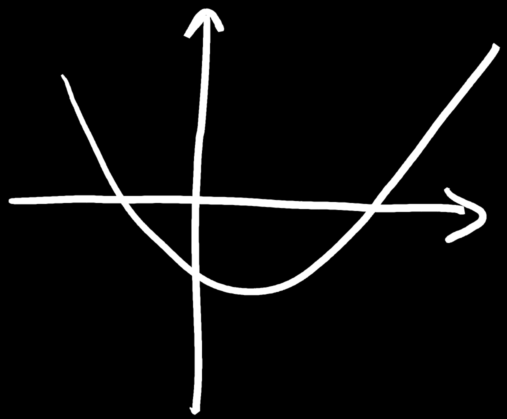
$h = \frac{\sqrt{30}}{2}$

I $y = ax + b$

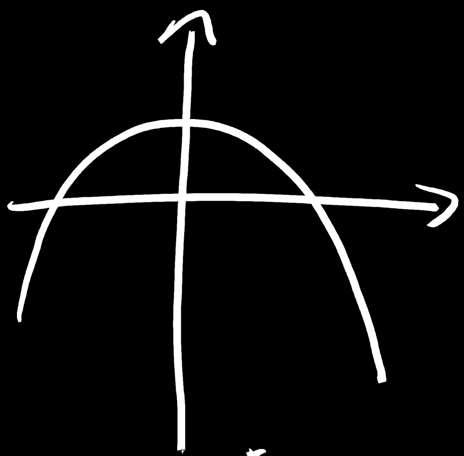


II $y = ax^2 + bx + c$

$a > 0$

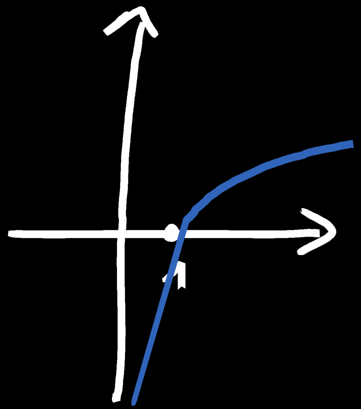


$a < 0$

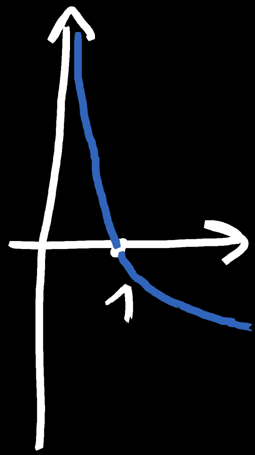


V $y = \log_a x$

$a > 1$

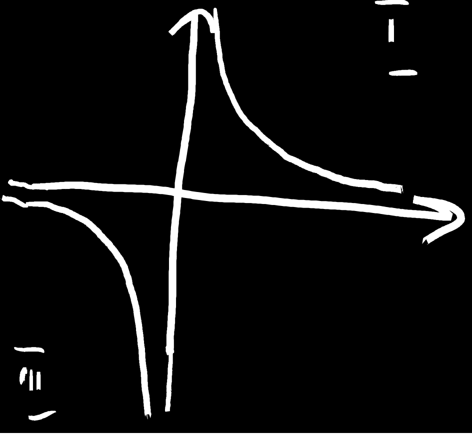


$a \in (0,1)$

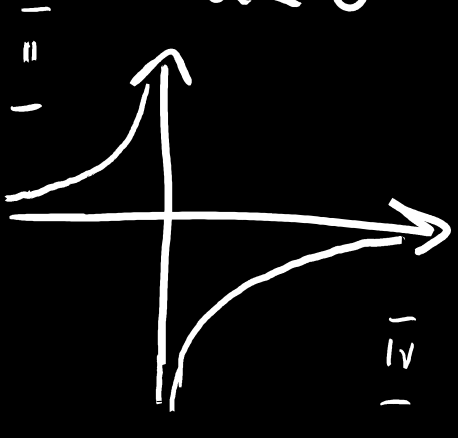


III $y = \dfrac{a}{x}$

$a > 0$

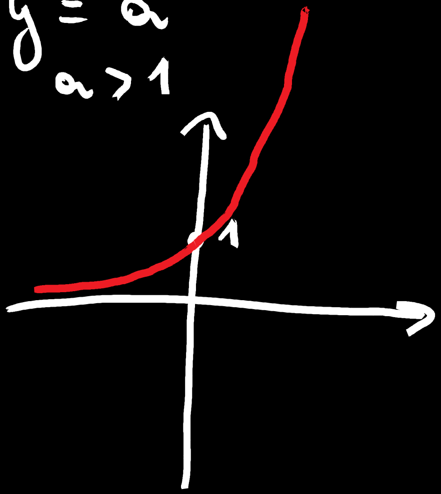


$a < 0$



IV $y = a^x$

$a > 1$



$a \in (0,1)$