

1. $27 : 3 = 9$

2 pkt

PUNKTACJA

0 - 7,5 mdst

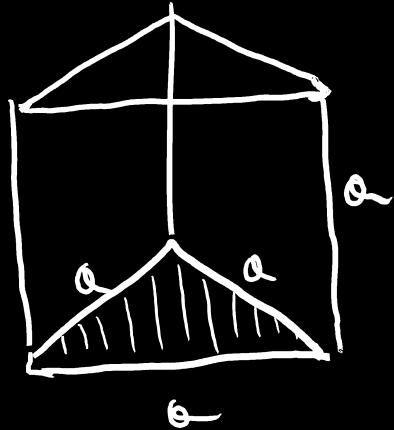
8 - 9,5 dop

10 - 13,5 dst

14 - 17,5 db

18 - 20 bdb

2.

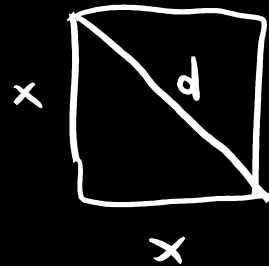
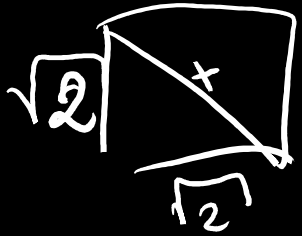
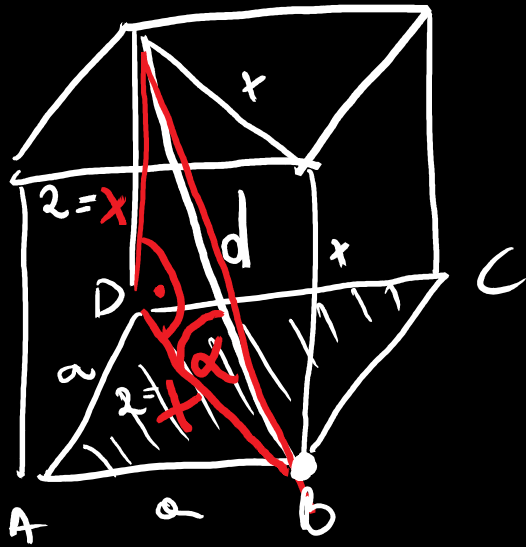


$$P_p = \frac{a^2 \sqrt{3}}{4} \quad 0,5 \text{ pkt}$$

$$P_b = 3a^2 \quad 0,5 \text{ pkt}$$

$$\frac{P_p}{P_b} = \frac{\frac{a^2 \sqrt{3}}{4}}{\frac{3a^2}{1}} = \frac{\cancel{a^2} \sqrt{3}}{4} \cdot \frac{1}{\cancel{3a^2}} = \frac{\sqrt{3}}{12} \quad 1 \text{ pkt}$$

3.



$$P_{\square} = 2$$

$$\alpha = 45^{\circ}$$

$$x = a\sqrt{2}$$

$$x = \sqrt{2}\sqrt{2} = 2 \quad 0,5 \text{ plit}$$

$$P_{\square} = a^2$$

$$a^2 = 2$$

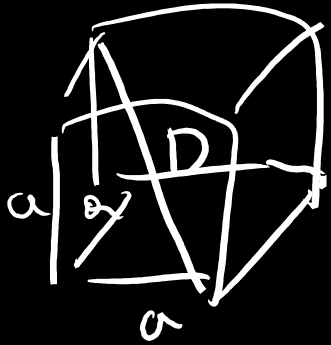
$$a = \sqrt{2} \quad 0,5 \text{ plit}$$

$$d = x\sqrt{2}$$

$$d = 2\sqrt{2}$$

1 plit

4



$$D = a\sqrt{3}$$

$$D = 6\sqrt{2}$$

$$a\sqrt{3} = 6\sqrt{2} \quad / : \sqrt{3}$$

$$a = \frac{6\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{6}}{3} = 2\sqrt{6} \quad 1 \text{ pkt}$$

$$V = a^3$$

$$V = (2\sqrt{6})^3$$

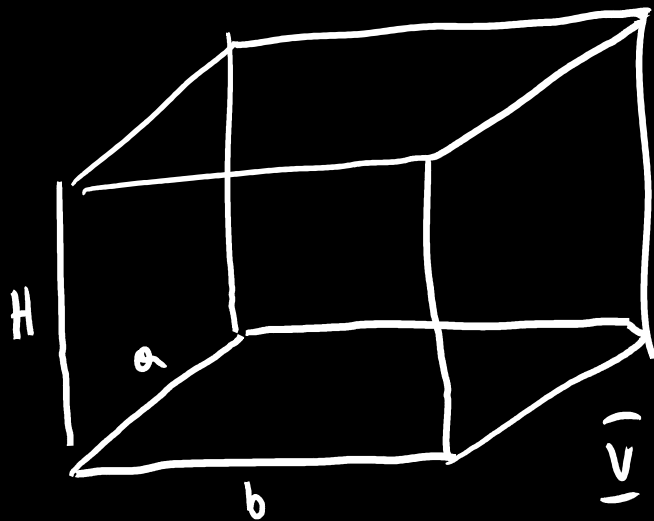
$$V = 8 \cdot 6 \cdot \sqrt{6}$$

$$V = 48\sqrt{6} \quad 1 \text{ pkt}$$

$$\sqrt{6} \cdot \sqrt{6} \cdot \sqrt{6}$$

5.

$V = ?$



$P_1 = 48 \text{ cm}^2$

$P_1 = a \cdot H$

$48 = a \cdot H \quad / : H$

$\frac{48}{H} = a$

$P_2 = 72 \text{ cm}^2$

$P_2 = b \cdot H$

$72 = b \cdot H \quad / : H$

$\frac{72}{H} = b$

$V = P_r \cdot H$

$V = 24 \cdot 6\sqrt{2} = 144\sqrt{2}$
0,5 pkt cm^3

$48 = a \cdot b$

$48 = \frac{48}{H} \cdot \frac{72}{H} \quad / : 48$

$1 = \frac{1}{H} \cdot \frac{72}{H}$

$H^2 = 72$

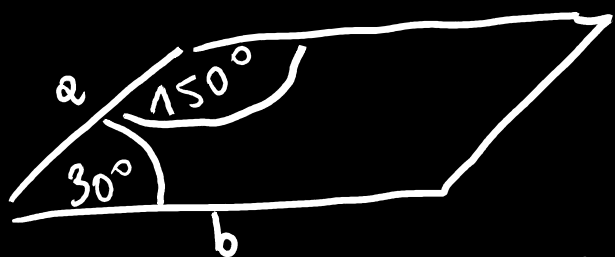
$1 = \frac{72}{H^2} \quad / \cdot H^2$

$H = \sqrt{36 \cdot 2}$

$H = 6\sqrt{2}$

0,5 pkt

I, II, III 1 pkt



$P_r = 24 \text{ cm}^2$

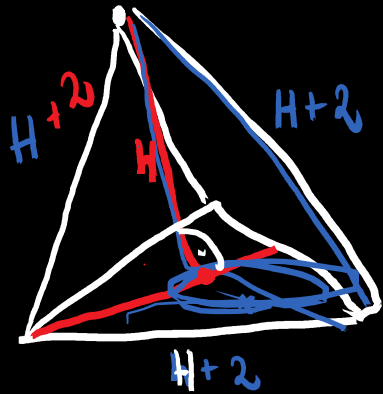
$P_r = a \cdot b \sin \alpha$

$24 = a \cdot b \sin 30^\circ$

$24 = a \cdot b \cdot \frac{1}{2} \quad / \cdot 2$

$48 = a \cdot b$

6



$$a = H + 2 = 2 + 2 = 4$$

$$x = \frac{2}{3} h = \frac{2}{3} \cdot \frac{a\sqrt{3}}{2} = \frac{a\sqrt{3}}{3} = \frac{(H+2)\sqrt{3}}{3} \quad 0,5 \text{ pkt}$$

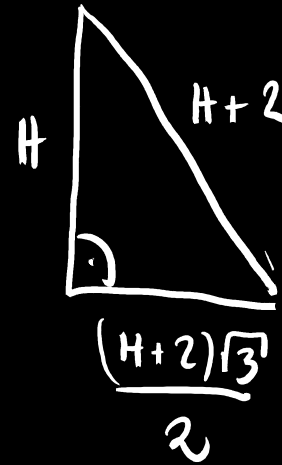
$$h = \frac{a\sqrt{3}}{2}$$

$$P_c = \frac{a^2 \sqrt{3}}{4} \cdot 4$$

$$P_c = \frac{4^2 \sqrt{3}}{4} \cdot 4$$

$$P_c = 16\sqrt{3} \quad 0,5 \text{ pkt}$$

$$P_c = ?$$



$$H^2 + \left[\frac{(H+2)\sqrt{3}}{2} \right]^2 = (H+2)^2$$

$$H^2 + \frac{(H+2)^2 \cdot 3}{4} = (H+2)^2$$

$$H^2 + \frac{3}{4}(H+2)^2 = (H+2)^2$$

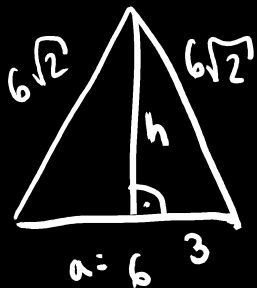
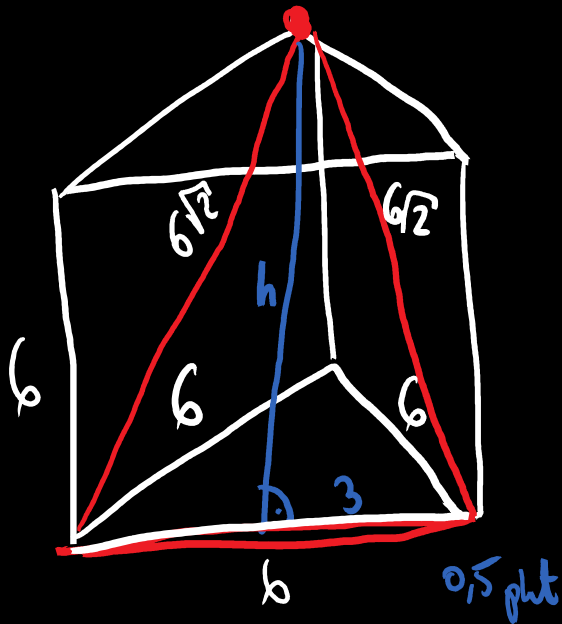
$$H^2 = \frac{1}{4}(H+2)^2 \quad 0,5 \text{ pkt}$$

$$H = \frac{1}{2}(H+2) \quad / \cdot 2$$

$$2H = H + 2$$

$$H = 2 \quad 0,5 \text{ pkt}$$

7



$$P = ?$$

$$P = \frac{1}{2} a h$$

$$3^2 + h^2 = (6\sqrt{2})^2 \quad 0,5 \text{ plit}$$

$$9 + h^2 = 36 \cdot 2$$

$$9 + h^2 = 72$$

$$h^2 = 63$$

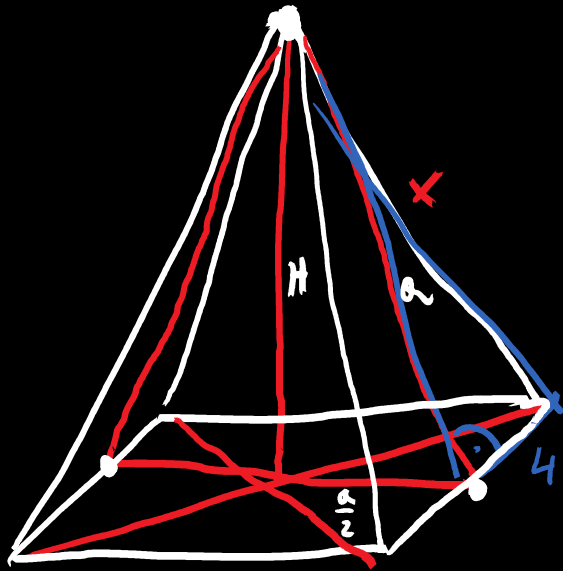
$$h = \sqrt{9 \cdot 7}$$

$$h = 3\sqrt{7} \quad 0,5 \text{ plit}$$

$$P = \frac{1}{2} \cdot 6 \cdot 3\sqrt{7}$$

$$P = 9\sqrt{7} \quad 0,5 \text{ plit}$$

8



$$P = 16\sqrt{3}$$

$$\text{I} \quad P = \frac{1}{2} a \cdot H$$

$$16\sqrt{3} = \frac{1}{2} a \cdot H \quad | \cdot 2$$

$$32\sqrt{3} = a \cdot H \quad | : a$$

$$\frac{32\sqrt{3}}{a} = H \quad 0,5 \text{ pkt}$$

$$\left(\frac{a}{2}\right)^2 + H^2 = a^2$$

$$\frac{a^2}{4} + H^2 = a^2$$

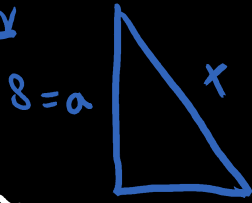
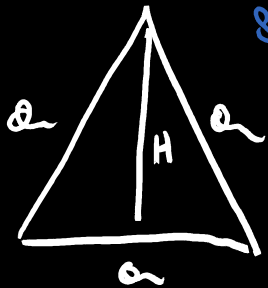
$$H^2 = a^2 - \frac{1}{4}a^2$$

$$80 = x^2 \quad x = \sqrt{16 \cdot 5} = 4\sqrt{5} \quad 0,5 \text{ pkt}$$

$$\text{III} \quad \frac{32\sqrt{3}}{a} = \frac{\sqrt{3}a}{2}$$

$$a^2 = 64$$

$$a = 8 \quad 0,5 \text{ pkt}$$



$$8^2 + 4^2 = x^2$$

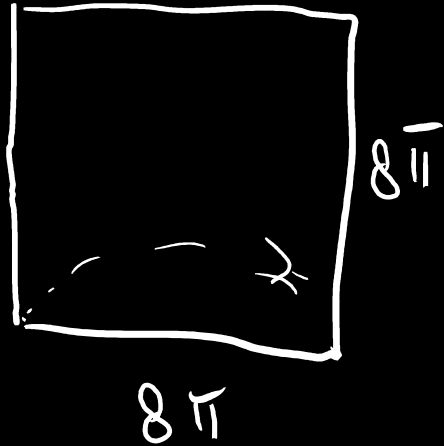
$$64 + 16 = x^2$$

$$80 = x^2 \quad x = \sqrt{16 \cdot 5} = 4\sqrt{5} \quad 0,5 \text{ pkt}$$

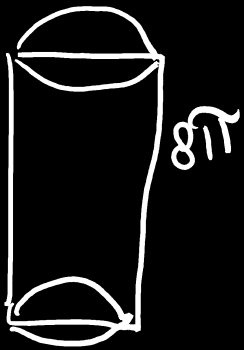
$$\text{II} \quad H^2 = \frac{3}{4} a^2 \quad | \sqrt{\quad}$$

$$H = \frac{\sqrt{3}a}{2} \quad 0,5 \text{ pkt}$$

9.



$$P_b = (8\pi)^2 = 64\pi^2 \quad 0,5 \text{ pt}$$



$$V_w = ?$$

$$P_b = 2\pi r \cdot H \quad 0,5 \text{ pt}$$

$$64\pi^2 = 2\pi \cdot r \cdot 8\pi$$

$$64\pi^2 = 16\pi^2 \cdot r \quad / : 16\pi^2$$

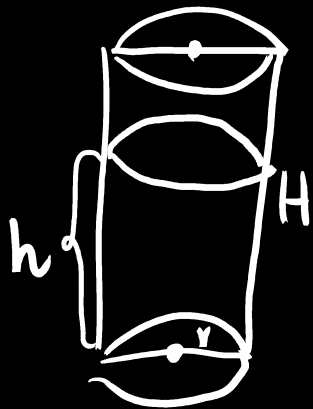
$$4 = r \quad 0,5 \text{ pt}$$

$$V_w = \pi r^2 \cdot H = \pi \cdot 4^2 \cdot 8\pi = 128\pi^2 \quad 0,5 \text{ pt}$$

$$10. \text{ Obw} = 40 \text{ cm} = 4 \text{ dm}$$

$$H = 50 \text{ cm} = 5 \text{ dm}$$

$$V = 3 \text{ dm}^3$$



$$\text{Obw} = 2\pi r$$

$$4 = 2\pi r \quad /: 2\pi$$

$$\frac{4}{2\pi} = r$$

$$\frac{2}{\pi} = r \quad \text{1 plit}$$

$$V = \pi r^2 \cdot h$$

$$3 = \pi \cdot \left(\frac{2}{\pi}\right)^2 \cdot h$$

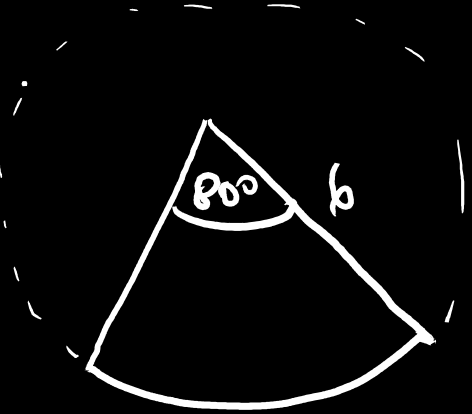
$$3 = \cancel{\pi} \cdot \frac{4}{\cancel{\pi}^2} \cdot h$$

$$3 = \frac{4}{\pi} \cdot h \quad /: \frac{4}{\pi}$$

$$\frac{3 \cdot \pi}{4} = h$$

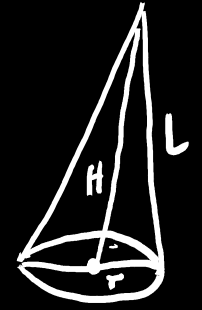
1 plit

11



~~$$P_b = \pi r L$$

$$L = b$$~~



~~$$\pi \cdot b^2 \cdot 360^\circ$$

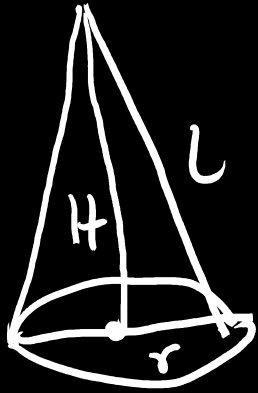
$$P_b = 80^\circ$$~~

1 plit $\pi \cdot 36 \cdot 80^\circ = P_b \cdot 360^\circ$

$$\frac{\pi \cdot 36 \cdot 80^\circ}{360^\circ} = P_b$$

1 plit $8\pi = P_b$

12



$$V = \frac{1}{3} \pi r^2 H$$

$$l = a = 6$$

$$r = \frac{a}{2} = 3$$

0,5 pkt

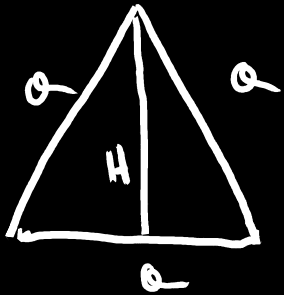
$$V = ?$$

I =

$$H = \frac{a\sqrt{3}}{2}$$

0,5 pkt

$$H = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$



$$P_{\Delta} = 9\sqrt{3}$$

I =

$$P_{\Delta} = \frac{a^2\sqrt{3}}{4}$$

0,5 pkt

$$9\sqrt{3} = \frac{a^2\sqrt{3}}{4} \quad | \cdot 4 \quad | \sqrt{3}$$

$$36 = a^2 \quad | \sqrt{\quad}$$

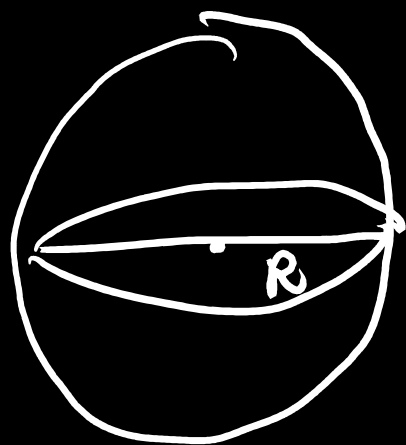
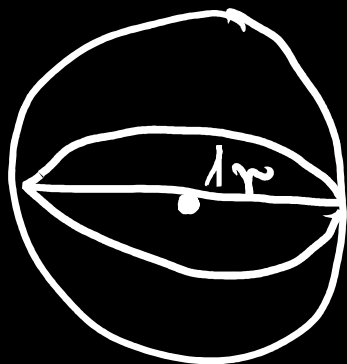
$$6 = a$$

$$III \quad V = \frac{1}{3} \pi \cdot 3^2 \cdot 3\sqrt{3}$$

$$V = 9\sqrt{3} \pi$$

0,5 pkt

13



$$k = \frac{R}{r} = \frac{1,6r}{r}$$

$$k = 1,6 \quad /(\)^2$$

$$k^2 = 2,56$$

$$k^2 = \frac{P_2}{P_1} \quad 0,5 \text{ pkt}$$

$$\bar{1} \quad R = n + 60\% v$$

$$R = 160\% v$$

$$R = 1,6 v \quad 0,5 \text{ pkt}$$

$$\rightarrow 2,56 = \frac{P_2}{P_1} / P_1$$

$$2,56 P_1 = P_2$$

$$0,5 \text{ pkt} \quad 256\% P_1 = P_2$$

Powierzchnia kuli
wzrosła o 156%
0,5 pkt

$$14. \text{I } d_1 = 4 \quad r_1 = 2 \quad V_1 + V_2 + V_3 = V_p$$

P_c pot' kul'i

$$d_2 = 8 \quad r_2 = 4$$

$$\text{II} \quad V_p = \frac{32}{3} \pi + \frac{256}{3} \pi + \frac{864}{3} \pi$$

$$d_3 = 12 \quad r_3 = 6$$

$$\text{0.5 pt} \quad V_p = \frac{1152}{3} \pi = 384 \pi$$



$$\text{V} \quad P_c = \frac{4\pi R^2}{2} + \pi R^2$$

$$\text{IV} \quad V_p = \frac{4}{3} \pi R^3 \cdot \frac{1}{2}$$

$$P_c = 3\pi R^2 = 3\pi \sqrt[3]{576^2} \quad \text{0.5 pt}$$

$$384\pi = \frac{2}{3} \pi R^3 \quad /: \pi$$

$$384 = \frac{2}{3} R^3 \quad / \cdot \frac{3}{2}$$

$$\overset{192}{384} \cdot \frac{3}{2} = R^3$$

$$576 = R^3 \quad / \sqrt[3]{\quad}$$

$$\sqrt[3]{576} = R \quad \text{0.5 pt}$$

$$\text{I} \quad V_1 = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \cdot 2^3 = \frac{4}{3} \pi \cdot 8 = \frac{32}{3} \pi$$

$$V_2 = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \cdot 4^3 = \frac{4}{3} \pi \cdot 64 = \frac{256}{3} \pi$$

$$V_3 = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \cdot 6^3 = \frac{4}{3} \pi \cdot 216 = \frac{864}{3} \pi \quad \text{0.5 pt}$$

15.

$$k^3 = \frac{V_1}{V_2} = \frac{1}{64} \sqrt[3]{\quad}$$

$$k = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$$

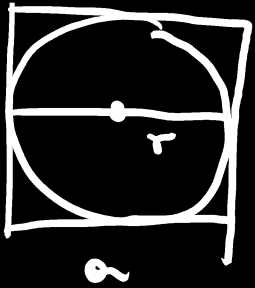
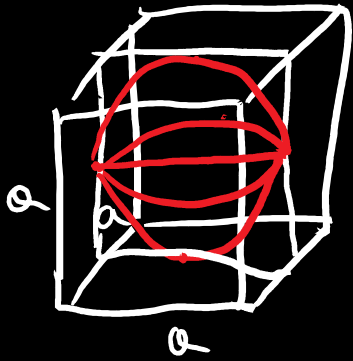
1 pt

$$k^2 = \frac{P_1}{P_2} = ?$$

$$\frac{P_1}{P_2} = k^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

1 pt

16



$$a = 2r$$

$$\underline{V_L = 32\pi}$$

$$V_{S2} = ?$$

$$\underline{V_L = \frac{4}{3}\pi r^3}$$

$$V_{S2} = a^3$$

$$V_{S2} = (2r)^3 = 8r^3$$

$$V_{S2} = 8 \cdot 24 = 192$$

1 plit

$$32\pi = \frac{4}{3}\pi r^3 \quad / \cdot \frac{3}{4} : \pi$$

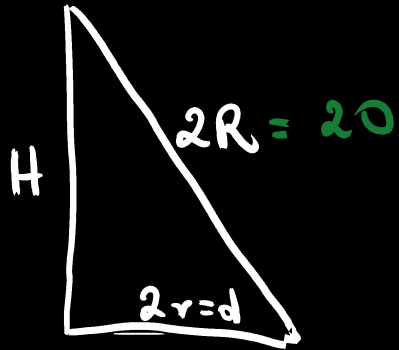
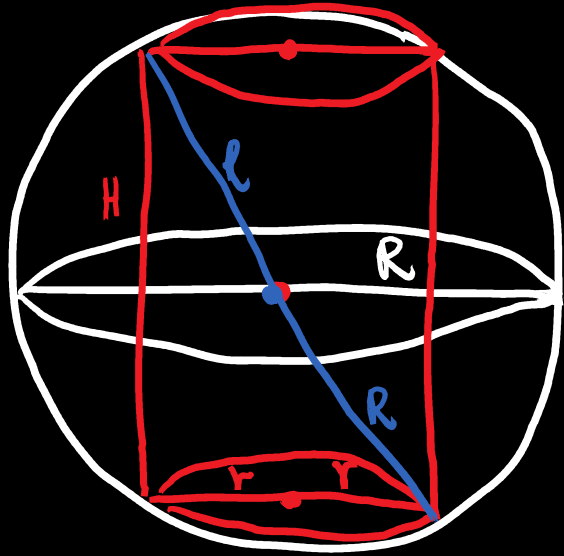
$$8 \cdot \cancel{3} \cdot \frac{\cancel{3}}{4} = r^3$$

$$\sqrt[3]{24 = r^3} \quad / \sqrt[3]{}$$

$$\sqrt[3]{8 \cdot 3} = r$$

$$2^3 \sqrt[3]{3} = r \quad 1 \text{ plit}$$

17.



$$P_k = 400\pi$$

$$V_w = ?$$

$$\text{i)} \quad \frac{H}{d} = \sqrt{3} \quad | \cdot d$$

0,5 pt

$$H = \sqrt{3} d = 10\sqrt{3}$$

$$\text{ii)} \quad P_k = 4\pi R^2$$

$$400\pi = 4\pi R^2 \quad | : \pi \cdot 4$$

$$\text{iii)} \quad H^2 + d^2 = 20^2$$

$$(\sqrt{3}d)^2 + d^2 = 400$$

$$3d^2 + d^2 = 400$$

$$100 = R^2$$

0,5 pt

$$10 = R$$

$$\text{iv)} \quad V_w = \pi r^2 \cdot H$$

$$4d^2 = 400 \quad | : 4$$

$$d = 2r$$

$$V_w = \pi \cdot 5^2 \cdot 10\sqrt{3}$$

$$d^2 = 100$$

$$r = 5$$

$$V_w = 250\sqrt{3}\pi$$

$$d = 10$$

0,5 pt

0,5 pt