

$$1. \log_{\sqrt{7}} \frac{49}{\sqrt{7}} = c = 3$$

$$\Rightarrow \sqrt{7}^c = \frac{49}{\sqrt{7}}$$

$$\left(7^{\frac{1}{2}}\right)^c = \frac{7^2}{7^{\frac{1}{2}}}$$

$$7^{\frac{1}{2}c} = 7^{\frac{3}{2}}$$

$$\frac{1}{2}c = \frac{3}{2} \quad | \cdot 2$$

$$c = 3$$

$$\log_a b = c \quad a^c = b$$

$$\begin{aligned} \Rightarrow \log_{\sqrt{7}} \frac{49\sqrt{7}}{7} &= \log_{\sqrt{7}} 7\sqrt{7} = \log_{\sqrt{7}} \sqrt{7}^2 \cdot \sqrt{7}^1 = \\ &= \log_{\sqrt{7}} \sqrt{7}^3 = 3 \end{aligned}$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\log_a a^x = x$$

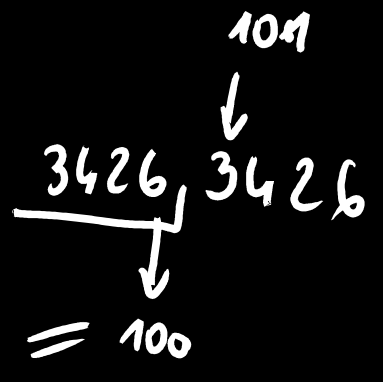
$$2 \quad \frac{5^3 \cdot \sqrt{5}}{5^{16}} \cdot \left(\frac{1}{5}\right)^{-15} = 5^{-13} \cdot 5^{\frac{1}{2}} \cdot 5^{15} = 5^{2\frac{1}{2}}$$

(B)

$$\frac{a^u}{a^m} = a^{u-m}$$

3.

$$5, \overset{4 \text{ digits}}{(3426)} \approx 5, \underbrace{3426}_{101} \underbrace{3426}_{101} 3426 \dots$$



(B)

4.

$$\frac{7}{x - \sqrt{x}} \cdot \frac{x + \sqrt{x}}{x}$$

$$x \in \mathbb{R}_+ \setminus \{1\}$$

$$7x = (x - \sqrt{x}) (x + \sqrt{x})$$

$$7x = x^2 - x$$

$a^2 - b^2$

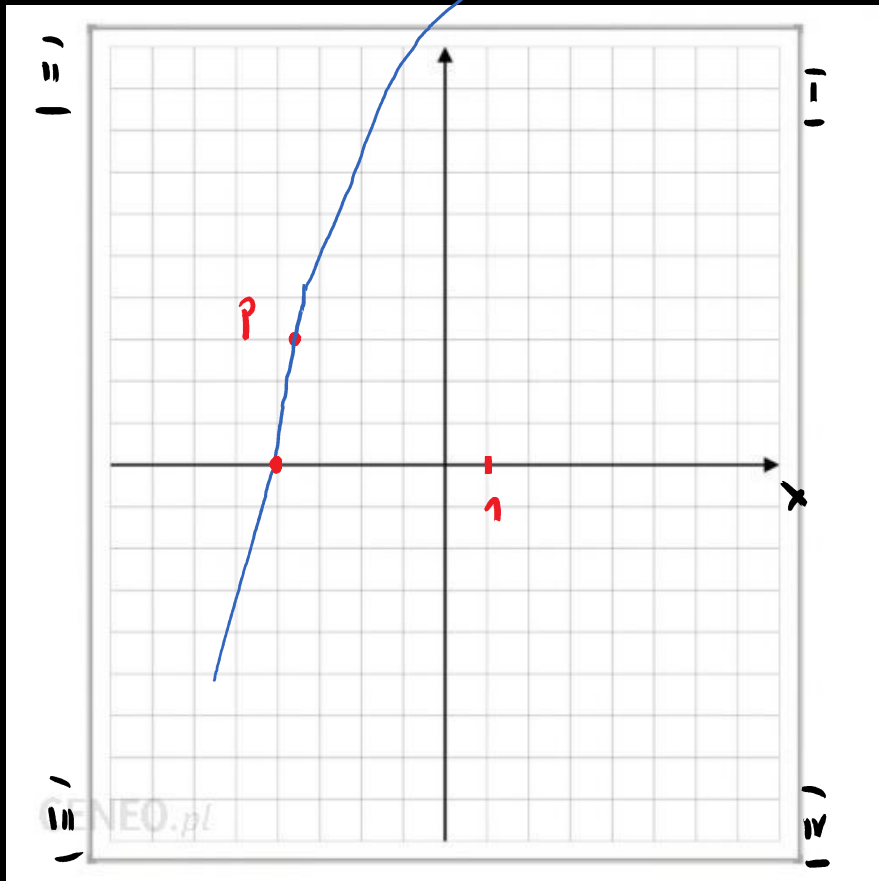
$$8x - x^2 = 0$$

$$x \cdot (8 - x) = 0$$

$$\begin{array}{c} \Downarrow \\ \boxed{x=0} \\ \text{SpN} \end{array} \quad \begin{array}{c} \Downarrow \\ \boxed{x=8} \end{array}$$

©

5. $P(-2\sqrt{3}, 3)$
 $-3,4$



$$m_0 = -\frac{7}{2}$$

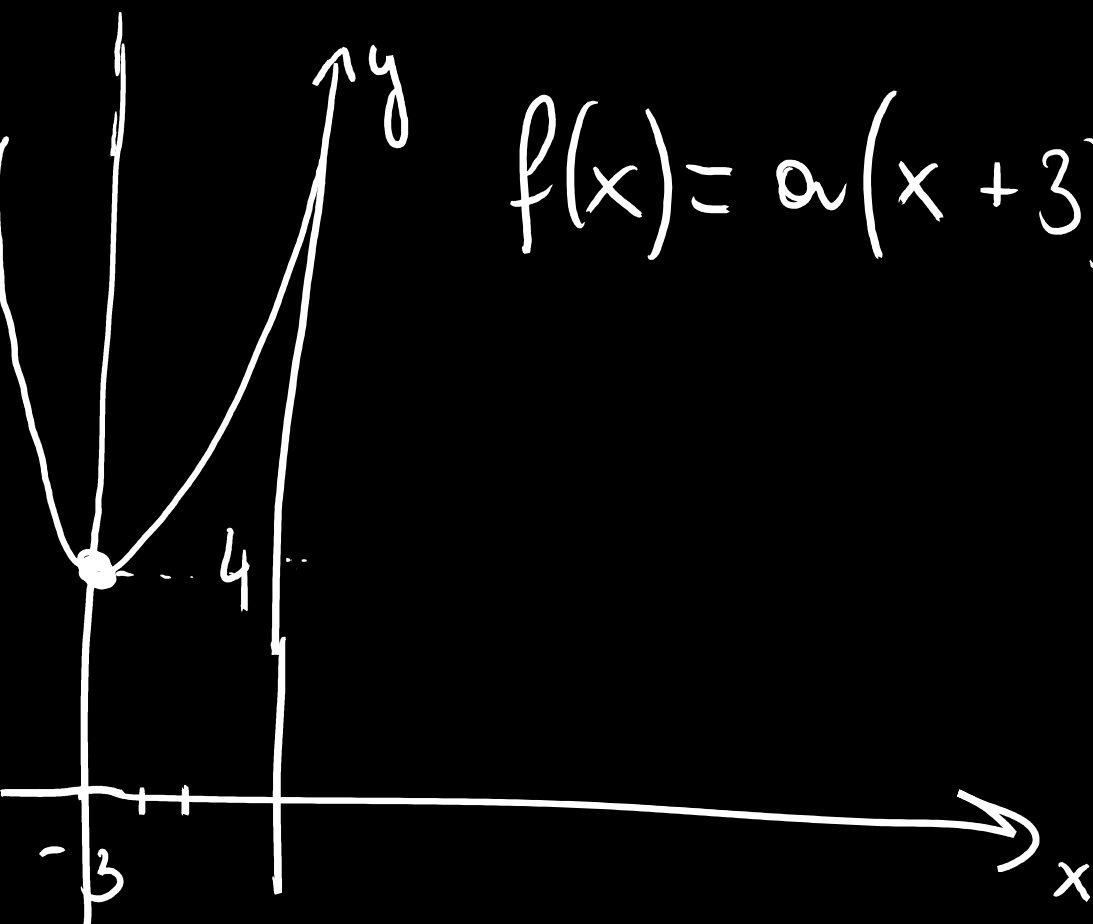
$$(x, y)$$
$$\left(-\frac{7}{2}, 0\right)$$

I, II, III

A

$$6. f(x) = a(x-p)^2 + q$$

$$W(p, q)$$



$$f(x) = a(x+3)^2 + 4$$

$$W(-3, 4)$$

↓ ↓
p q

(D)

7

$$\begin{aligned} a_1 &= 10 \\ a_2 &= 13 \\ a_3 &= 16 \end{aligned}$$

$$r = 3$$

$$n = 32$$

$$a_n = a_1 + (n-1)r$$

$$S_{32} = ?$$

$$S_n = \frac{2a_1 + (n-1)r}{2} \cdot n$$

$$S_{32} = \frac{2 \cdot 10 + (32-1) \cdot 3}{2} \cdot 32 = (20 + 31 \cdot 3) \cdot 16 =$$

$$= 113 \cdot 16 = 1808$$

(A)

$$8. \quad a_1 = \frac{200}{3}$$

$$a_3 = 0,(\overline{6})$$

$$a_5 = ?$$

$$\boxed{0,(\overline{6})} \approx \boxed{0,6666\dots} = \underline{x} / \cdot 10$$

$$= \frac{6}{3} \quad 6,666\dots = 10x$$

$$6 + \boxed{0,666\dots} = 10x$$

$$6 + x = 10x$$

$$6 = 9x \quad /: 9$$

$$\frac{6}{9} = x$$

$$\frac{2}{3} = x$$

$$0,0(\overline{6})$$

$$\frac{6}{90}$$

$$0,(\overline{56})$$

$$\frac{56}{99}$$

$$0,00(\overline{6})$$

$$\frac{6}{900}$$

$$0,0(\overline{56})$$

$$\frac{56}{990}$$

$$0,(\overline{125})$$

$$\frac{125}{999}$$

$$8. \quad a_1 = \frac{200}{3} \quad a_3 = 0,(\overline{6}) = \frac{2}{3} \quad a_5 = ?$$

$$\boxed{0,(\overline{6})} \approx \boxed{0,6666\dots} = x / \cdot 10$$

$$a_n = a_1 \cdot q^{n-1}$$

$$6,666\dots = 10x$$

$$a_5 = \frac{200}{3} \cdot \left(\frac{1}{10}\right)^4$$

$$6 + \boxed{0,666\dots} = 10x$$

$$a_5 = \frac{200}{3} \cdot \frac{1}{10000}$$

$$6 + x = 10x$$

$$a_5 = \frac{1}{150}$$

$$6 = 9x \quad | : 9$$

$$\frac{6}{9} = x$$

$$\frac{2}{3} = x$$

(D)

$$a_1 \cdot q^2 = a_3$$

$$\frac{200}{3} \cdot q^2 = \frac{2}{3} / \cdot \frac{3}{200}$$

$$q^2 = \frac{2}{3} \cdot \frac{3}{200} \quad 100$$

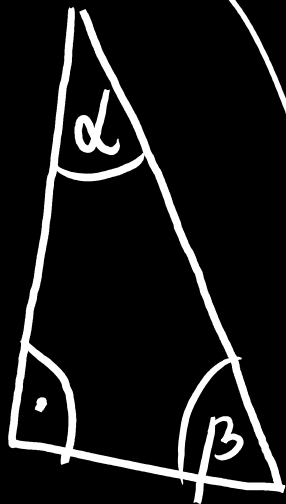
$$q^2 = \frac{1}{100}$$

$$\boxed{q = \frac{1}{10}}$$

$$q = -\frac{1}{10} \quad \text{Spur}$$

9

$$8 \sin^2 \alpha + \cos^2 \beta = 1$$



$$\cos \beta = \cos(90^\circ - \alpha) = \sin \alpha$$

$$8 \sin^2 \alpha + \sin^2 \alpha = 1$$

$$9 \sin^2 \alpha = 1 \quad | : 9$$

$$\sin^2 \alpha = \frac{1}{9}$$

$$\sin \alpha = \frac{1}{3}$$

$$\sin \alpha = -\frac{1}{3}$$

spn

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\frac{1}{9} + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = \frac{8}{9} \quad | \sqrt{\quad}$$

$$\cos \alpha = \frac{2\sqrt{2}}{3}$$

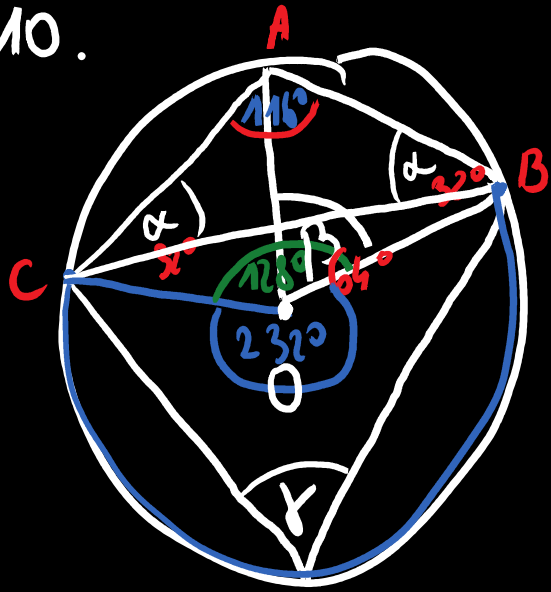
$$\tan \alpha = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{3} \cdot \frac{3}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\alpha + \beta = 90^\circ$$

$$\beta = 90^\circ - \alpha$$

(B)

10.



$$\alpha = 32^\circ$$

$$\beta = ? \quad \gamma = ?$$

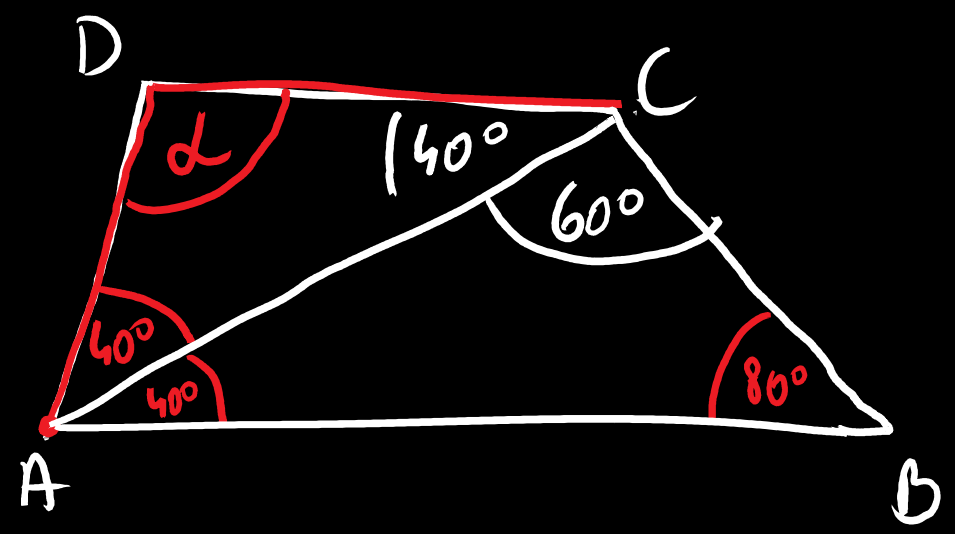
$$\beta = 64^\circ$$

$$360^\circ - 232^\circ = 128^\circ$$

$$\gamma = \frac{128^\circ}{2} = 64^\circ$$

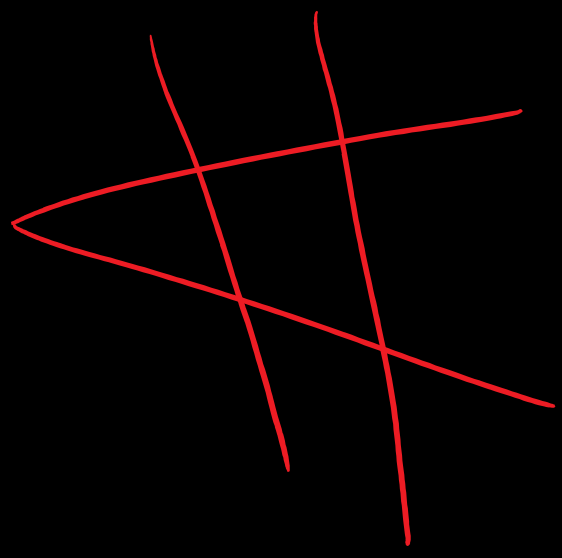
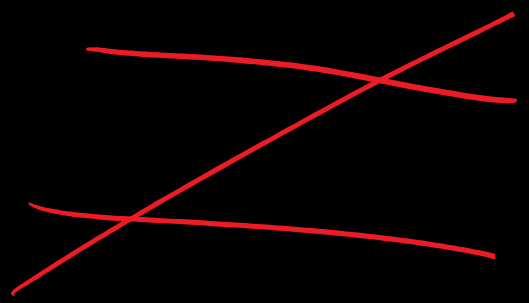
(B)

11.

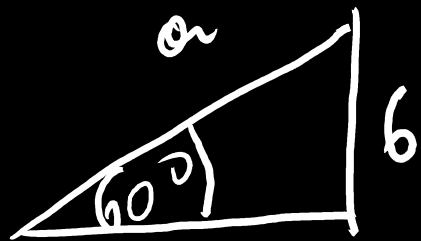
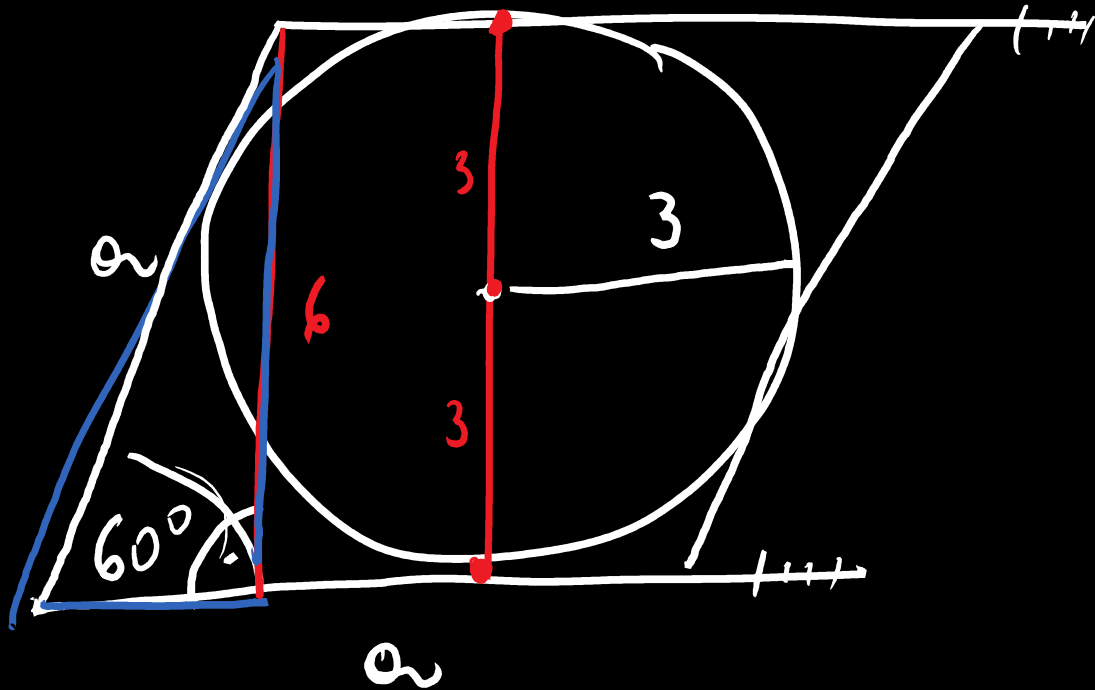


$AB \parallel CD$

$\angle ADC = \alpha = ?$
 \uparrow
 $\alpha = 100^\circ$



12.



(D)

$$h = 6$$

$$\sin \beta = \frac{b}{c} = \frac{6}{a}$$

$$\sin 60^\circ = \frac{6}{a}$$

$$\frac{\sqrt{3}}{2} = \frac{6}{a}$$

$$\sqrt{3} a = 12 \quad / \sqrt{3}$$

$$3a = 12\sqrt{3} \quad a = 4\sqrt{3}$$

$$13. \text{Zwei} \quad \underline{a} > 2b > \underline{1}$$

$$a > 2b$$

$$a - 2b > 0$$

$$a > 1$$

$$a - 1 > 0$$

$$(?) \quad \frac{a^2 - a}{2} > ab - b \quad | \cdot 2$$

$$a^2 - a > 2ab - 2b$$

$$a^2 - a - 2ab + 2b > 0$$

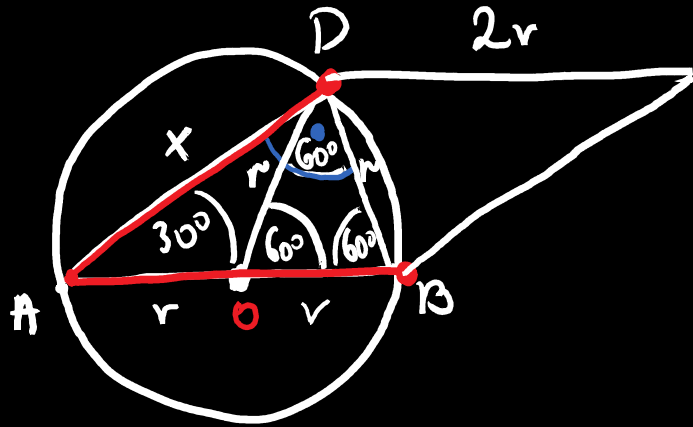
$$\underline{a^2 - 2ab} - a + 2b > 0$$

$$a \underline{(a - 2b)} - 1 \underline{(a - 2b)} > 0$$

$$\underbrace{(a - 2b)}_{> 0} \underbrace{(a - 1)}_{> 0} > 0$$

c. n. d

14.



$$|AB| = |CD| = 2r$$

$$a^2 + b^2 = c^2$$

$$x^2 + r^2 = (2r)^2$$

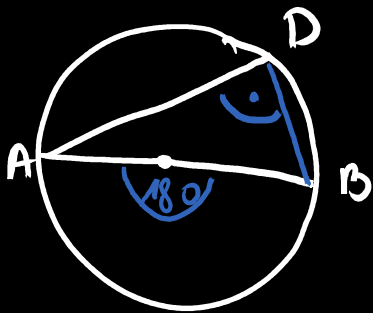
$$x^2 = 4r^2 - r^2$$

$$x^2 = 3r^2 \quad | \sqrt{\quad}$$

$$|AD| = x = \sqrt{3}r$$

$$(\text{?}) \frac{|AD|}{|AB|} = \frac{\sqrt{3}}{2} < 1$$

uniqueness
highline



$$\frac{|AD|}{|AB|} = \frac{\sqrt{3}r}{2r} = \frac{\sqrt{3}}{2}$$

c.u.d

15. $a_5 = 0$ c. anyfun. \rightarrow

$$S_4 = -130$$

$$-130 = \frac{2 \cdot (-4v) + 3v}{2} \cdot 4^2$$

$$-130 = -10v \quad / : (-10)$$

$$\boxed{13 = v}$$

$$b) \frac{2 \cdot (-52) + (n-1)13}{2} \cdot n \leq 143 \quad / \cdot 2$$

$$(-104 + 13n - 13)n \leq 286$$

$$-117n + 13n^2 - 286 \leq 0 \quad / 13$$

$$0 = a_1 + 4v$$

$$\boxed{a_1 = -4v = -52}$$

a) $v = ?$

b) $S_n \leq 143$

$$\underbrace{a_1, a_2, a_3, a_4}_{< 0}, \underbrace{a_5}_{= 0}, \underbrace{a_6, a_7}_{> 0}, \dots$$

$$-9n + n^2 - 22 \leq 0$$

$$S_n = \frac{2a_1 + (n-1)v}{2} \cdot n$$

$$a_n = a_1 + (n-1)v$$

$$a_5 = a_1 + 4v$$

$$-3n + n^2 - 22 \leq 0$$

$$n^2 - 3n - 22 \leq 0$$

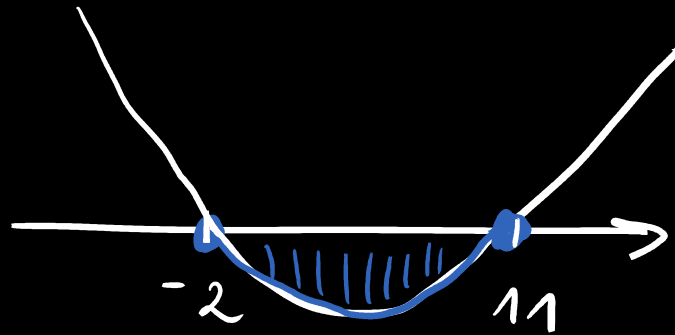
$$\Delta = 81 - 4 \cdot 1 \cdot (-22)$$

$$\Delta = 81 + 88 = 169$$

$$\sqrt{\Delta} = 13$$

$$n_1 = \frac{3 - 13}{2} = -2$$

$$n_2 = \frac{3 + 13}{2} = 11$$



$$n \in \langle -2, 11 \rangle$$

$$n \in \{1, 2, 3, \dots, 11\}$$

$$\max = 11$$

$$n \in \mathbb{N}$$

$$n \geq 1$$

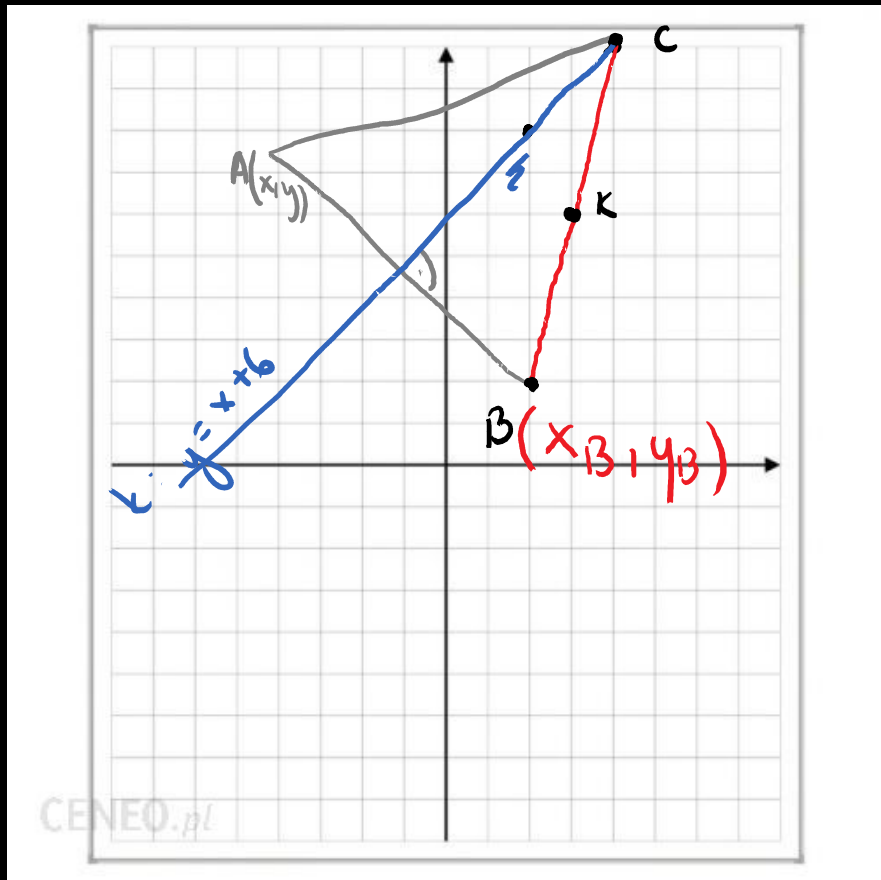
$$A(4, 10)$$

$$K(3, 6)$$

$$k: y = x + 6$$

$(4, 10) \quad (2, 8)$

$$A(x, y) = ?$$



$$AB \perp k$$

$$a_1 \cdot a_2 = -1$$

$$1 \cdot a_2 = -1$$

$$a_2 = -1$$

$$AB: y = -1x + b$$

$$K \left(\frac{4 + x_B}{2}, \frac{10 + y_B}{2} \right)$$

$$B(2, 2)$$

$$y = ax + b$$

$$3 = \frac{4 + x_B}{2}$$

$$6 = 4 + x_B$$

$$2 = x_B$$

$$6 = \frac{10 + y_B}{2}$$

$$12 = 10 + y_B$$

$$2 = y_B$$

16. $C(4, 10)$

$K(3, 6)$

$k: y = x + 6$

$A(x, y) = ?$

$B(2, 2)$

$(4, 10) (2, 8)$

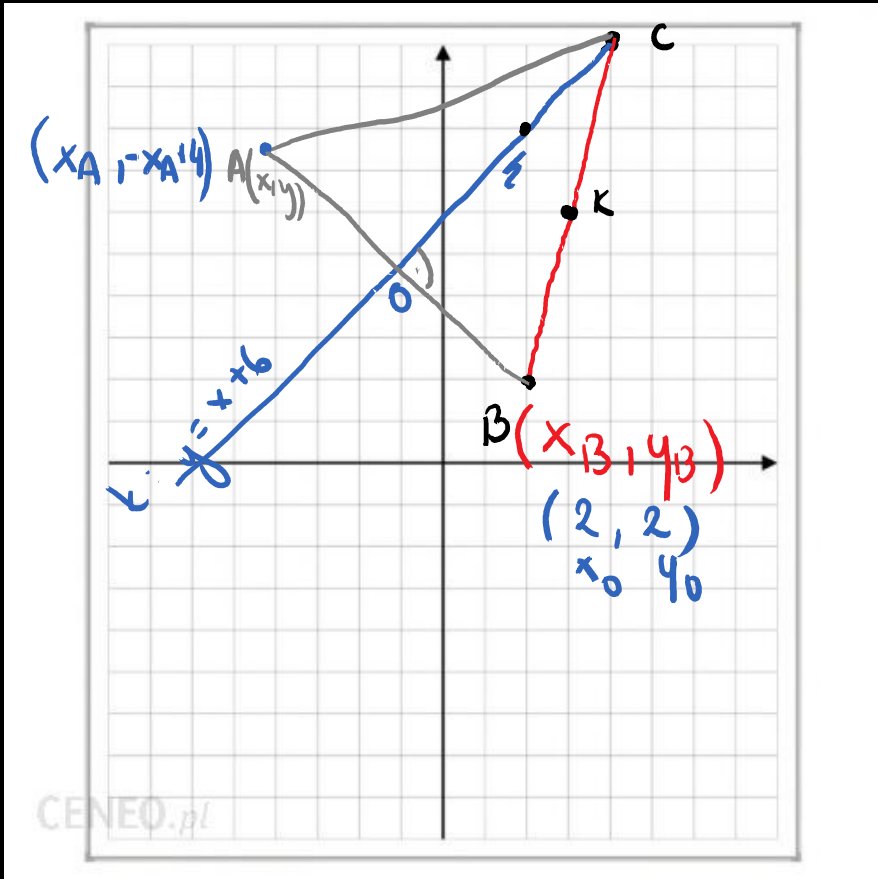
$k: y = x + 6$

$\Rightarrow y - x - 6 = 0$

$\Rightarrow -x + y - 6 = 0$

$\Rightarrow Ax + By + C = 0$

$A = -1 \quad B = 1 \quad C = -6$



AB: $y = -x + b$
 $2 = -1 \cdot 2 + b$
 $2 = -2 + b$
 $4 = b$

AB: $y = -x + 4$
 $y_A = -x_A + 4$

16. $C(4, 10)$

$K(3, 6)$

$$k: y = x + 6$$

$A(x, y) = ?$

$B(2, 2)$

$(4, 10) (2, 8)$

$$k: y = x + 6$$

$$\Rightarrow y - x - 6 = 0$$

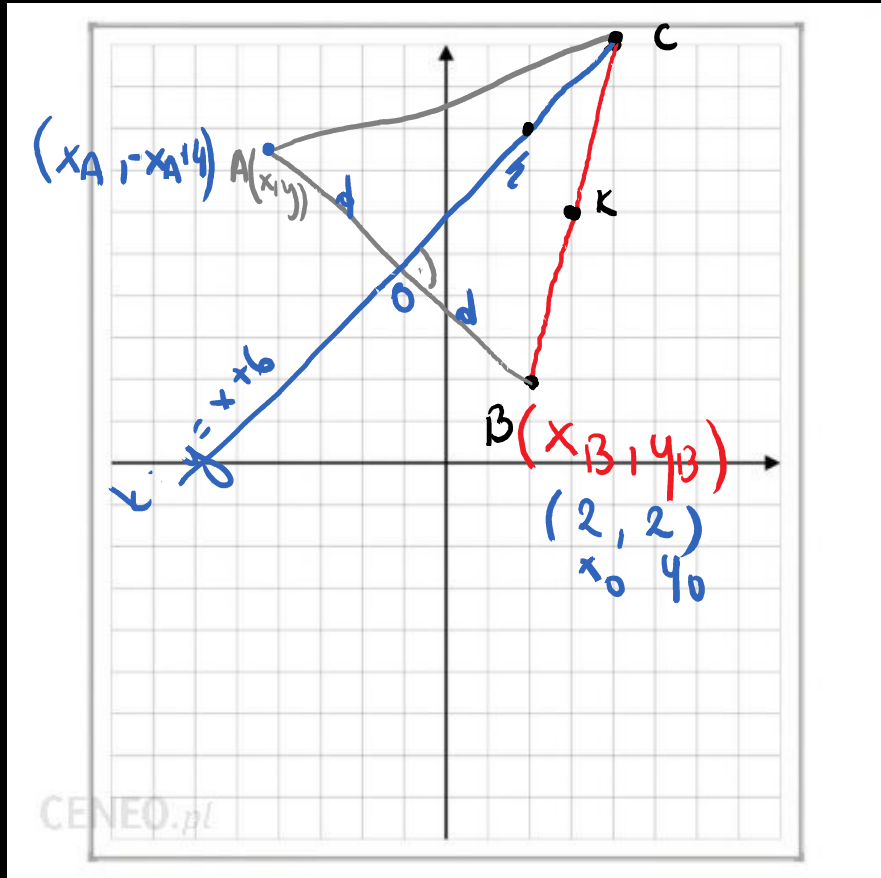
$$\Rightarrow -x + y - 6 = 0$$

$$\Rightarrow Ax + By + C = 0$$

$$A = -1 \quad B = 1 \quad C = -6$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{|-1 \cdot 2 + 1 \cdot 2 - 6|}{\sqrt{(-1)^2 + 1^2}} =$$

$$= \frac{|-2 + 2 - 6|}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$



$$A(4, 10)$$

$$K(3, 6)$$

$$k: y = x + 6$$

$$A(x, y) = ?$$

$$B(2, 2)$$

$$(4, 10) \quad (2, 8)$$

$$k: y = x + 6 \Rightarrow y - x - 6 = 0$$

$$\Rightarrow -x + y - 6 = 0$$

$$\Rightarrow Ax + By + C = 0$$

$$A = -1 \quad B = 1 \quad C = -6$$

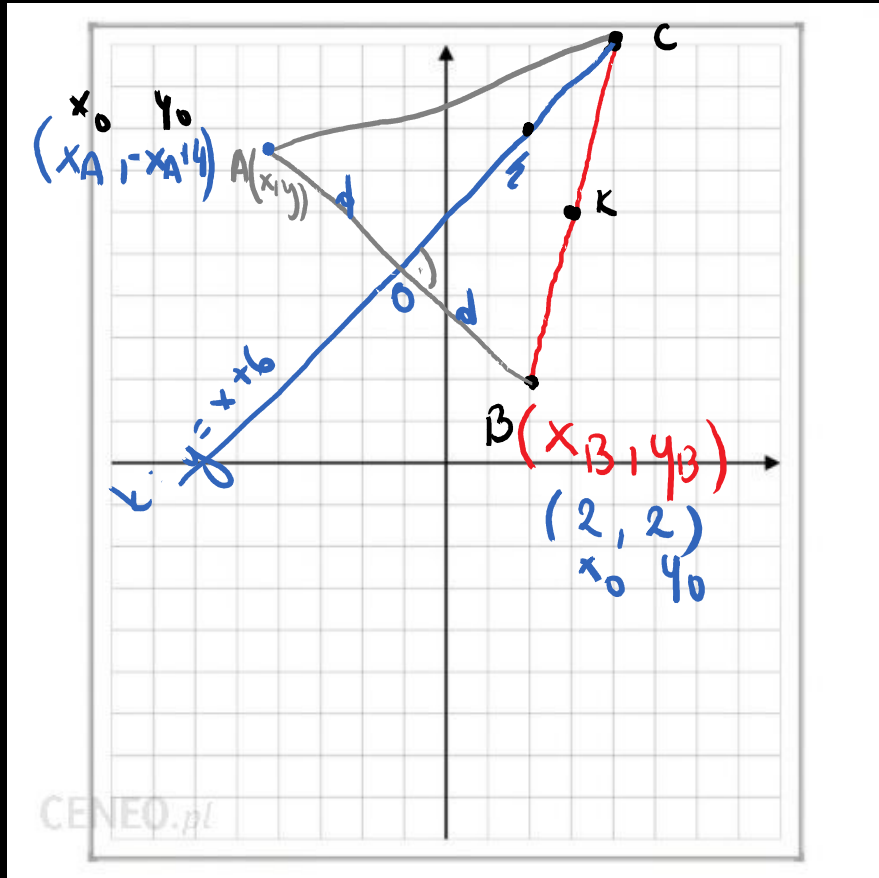
$$d = 3\sqrt{2}$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$3\sqrt{2} = \frac{|-1 \cdot x_A + 1 \cdot (-x_A + 4) - 6|}{\sqrt{2}} \quad / \cdot \sqrt{2}$$

$$6 = |-x_A - x_A + 4 - 6|$$

$$6 = |-2x_A - 2|$$



$$6 = |-2x_A - 2|$$

1°

$$-2x_A - 2 > 0$$

$$-2x_A > 2 \quad /: (-2)$$

$$x_A < -1$$

$$6 = -2x_A - 2$$

$$8 = -2x_A \quad /: (-2)$$

$$\underline{-4 = x_A}$$

$$y_A = -x_A + 4$$

$$y_A = 8 \quad \underline{A(-4, 8)}$$

2°

$$-2x_A - 2 < 0$$

$$x_A > -1$$

$$6 = 2x_A + 2$$

$$4 = 2x_A$$

$$2 = x_A$$

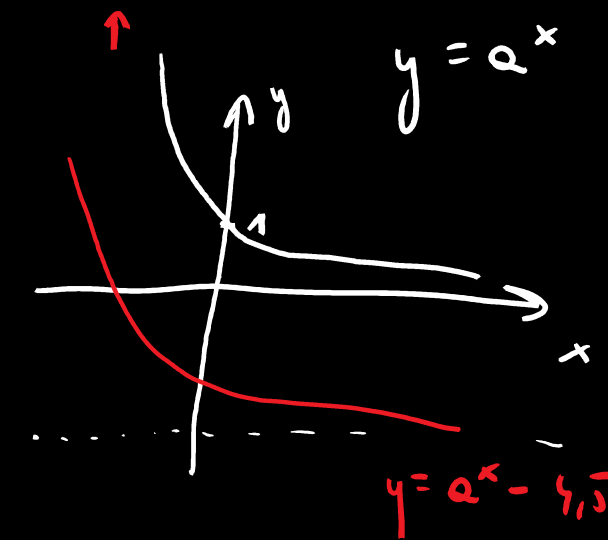
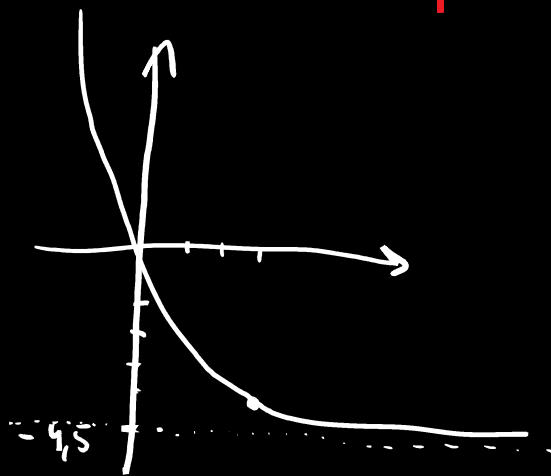
$$y_A = 2$$

$$B(2, 2)$$

$$3^6 \cdot 12^{-3} = 3^6 \cdot (4 \cdot 3)^{-3} = 3^6 \cdot 4^{-3} \cdot 3^{-3} = 3^3 \cdot \left(\frac{1}{4}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$17. \quad f(x) = a^{x-2} + b \quad x \in \mathbb{R}$$

$$\begin{matrix} x & f(x) \\ (3, -4) \end{matrix}$$



$$Zw = \left(-\frac{3}{2}, +\infty\right)$$

$$b = -4.5$$

$$f(x) = a^{x-2} - 4.5$$

$$-4 = a^{3-2} - 4.5$$

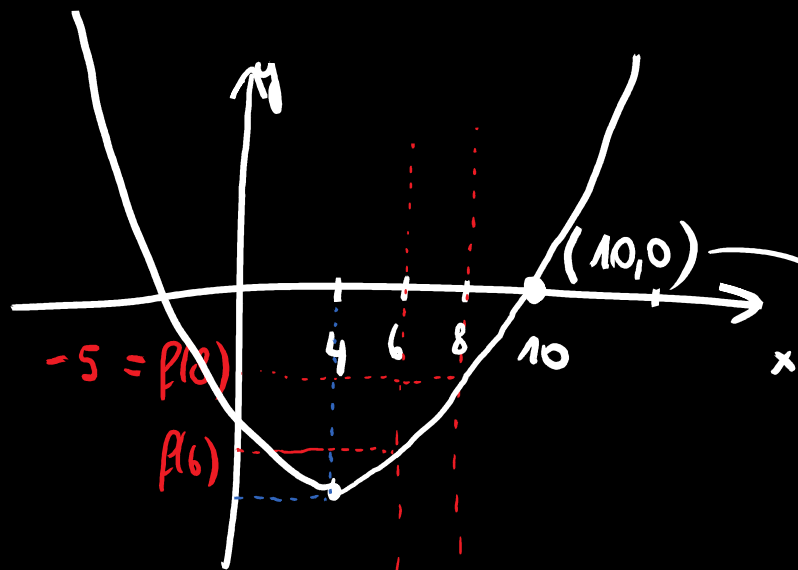
$$-4 = a - 4.5$$

$$0.5 = a$$

$$\boxed{f(x) = \left(\frac{1}{2}\right)^{x-2} - 4\frac{1}{2}}$$

18

$$m_0 = 10$$



$f \nearrow$ dla $x \in \langle 4, +\infty \rangle$

$f_{\max} = -5$ w przedziale $\langle 6, 8 \rangle$

$$f(x) = \frac{1}{4}x^2 - 2x - 5$$

$$p = 4$$

$$p = \frac{-b}{2a}$$

$$4 = \frac{-b}{2a} \quad | \cdot 2a$$

$$8a = -b$$

$$\underline{-8a = b}$$

$$-2 = b$$

$$\underline{f(x) = ax^2 + bx + c}$$

$$f(x) = ax^2 - 8ax + c$$

$$f(5) = a \cdot 8^2 - 8a \cdot 8 + c$$

$$0 = a \cdot 10^2 - 8a \cdot 10 + c$$

$$\left\{ \begin{array}{l} -5 = 64a - 64a + c \\ 0 = 100a - 80a + c \end{array} \right.$$

$$0 = 100a - 80a + c$$

$$\underline{c = -5}$$

$$f(8) = -5$$

$$f(x) = y$$

$$(8, -5)$$

$$0 = 20a - 5$$

$$\frac{5}{20} = a$$

$$5 = 20a \quad | : 20$$

$$\frac{1}{4} = a$$





